Neural Language Model

CSE 447 / 517 January 20th, 2022 (Week 3)

Eisenstein (2019) 6 and Appendix A

Logistics

- CSE 517 project proposal is due on Friday 1/21

Agenda

- Quiz 2 Solutions
- Feedforward Neural Network
- Convolutional Neural network
- Q&A

Quiz 2 - Problem Setup

You wanted to take a class but you were not sure about the workload. You then asked some friends who took the class. They told you the time they spent and the GPA they got in this class, which are in the following table:

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

Based on the table, fill in the blanks. Round answer to 1 decimal place if not specified otherwise.

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If you randomly ask one of the friends above, what is the probability that the person got 3.8?

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If you randomly ask one of the friends above, what is the probability that the person got 3.8?

$$p(X = 3.8) = \frac{4}{9} \approx 0.4$$

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If you randomly ask two of the friends above, what is the probability that they both got 4.0? (Use simplified fraction)

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If you randomly ask two of the friends above, what is the probability that they both got 4.0? (Use simplified fraction)

$$p(X_1 = 4.0, X_2 = 4.0) = p(X_1 = 4.0) \cdot p(X_2 = 4.0 \mid X_1 = 4.0)$$

$$= \frac{3}{9} \cdot \frac{2}{8}$$

$$= \frac{1}{12}$$

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

What is the probability of getting 4.0 if you spent 12 hours per week?

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

What is the probability of getting 4.0 if you spent 12 hours per week?

$$egin{aligned} p(X=4.0\,|\,H=12) &= rac{p(X=4.0,\,H=12)}{p(H=12)} \ &= rac{rac{2}{9}}{rac{4}{9}} \ &= 0.5 \end{aligned}$$

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

What is the probability of getting 3.9 or above if you spent 10 hours per week?

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

What is the probability of getting 3.9 or above if you spent 10 hours per week?

$$egin{aligned} p(X \geq 3.9 \,|\, H = 10) &= rac{p(X \geq 3.9,\, H = 10)}{p(H = 10)} \ &= rac{rac{2}{9}}{rac{5}{9}} \ &= 0.4 \end{aligned}$$

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If one of your friends got 3.9 in this class, what is the probability that your friend spent 12 hours in this class?

3.8	3.8	3.8	3.8	3.9	3.9	4.0	4.0	4.0
10	10	10	12	10	12	10	12	12

If one of your friends got 3.9 in this class, what is the probability that your friend spent 12 hours in this class?

$$p(H=12 \,|\, X=3.9) = rac{p(H=12,X=3.9)}{p(X=3.9)} = rac{rac{1}{9}}{rac{2}{9}} = 0.5$$

Quiz 2 - Problem Setup

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

We have trained a Bi-gram model based on some data. We store the frequencies of each pair of words in the following table. Each cell represents the occurrences of the top row word following right after the left column word. For example, "finished" appeared after "I" 40 times.

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I finished ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I finished work

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

Is the work ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

Is the work finished

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I saw ____ ___

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I saw the ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I saw the beautiful ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

I saw the beautiful gift

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

the ____ ___

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

the beautiful ____ ___

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

the beautiful gift ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

the beautiful gift finished ____

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

Please fill in the most likely word based on the previous word and the frequency table for the following sentences:

the beautiful gift finished work

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	0	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

What is the probability that "work" appears after "finished"? (Use simplified fraction)

1st/2nd	I	finished	work	saw	the	beautiful	gift
1	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	U	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

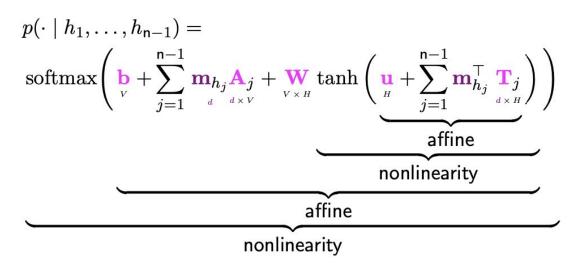
What is the probability that "work" appears after "finished"? (Use simplified fraction)

1st/2nd	I	finished	work	saw	the	beautiful	gift
I	0	40	20	30	0	0	0
finished	0	0	10	0	8	5	3
work	0	5	U	0	0	0	0
saw	0	5	5	0	8	5	3
the	0	10	15	0	0	20	10
beautiful	0	0	5	0	0	0	10
gift	0	5	0	0	0	3	0

What is the probability that "work" appears after "finished"? (Use simplified fraction)

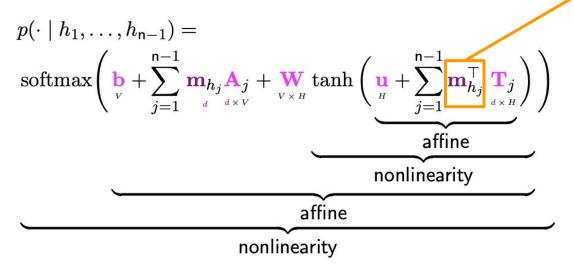
$$p(X_2 = \text{work} \,|\, X_1 = \text{finished}) = \frac{10}{10 + 8 + 5 + 3} = \frac{5}{13}$$

Define the n-gram probability as follows:



Parameters θ include M and everything in pink.

Define the n-gram probability as follows:



Embedding of history token h_j.

Parameters θ include M and everything in pink.

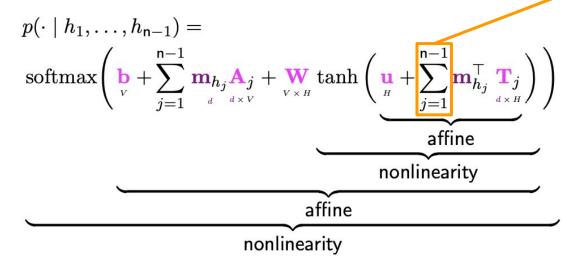
Define the n-gram probability as follows:

$$p(\cdot \mid h_1, \dots, h_{\mathsf{n}-1}) = \\ \operatorname{softmax} \left(\underbrace{\begin{smallmatrix} \mathbf{b} \\ \mathbf{b} \end{smallmatrix}}_{V} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j} \underbrace{\begin{smallmatrix} \mathbf{A} \\ \mathbf{d} \times V \end{smallmatrix}}_{d \times V} + \underbrace{\begin{smallmatrix} \mathbf{W} \\ \mathbf{V} \times H \end{smallmatrix}}_{V \times H} \operatorname{tanh} \left(\underbrace{\begin{smallmatrix} \mathbf{u} \\ \mathbf{u} \end{smallmatrix}}_{H} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}^{\top} \underbrace{\begin{smallmatrix} \mathbf{T} \\ \mathbf{d} \times H \end{smallmatrix}}_{d \times H} \right) \right) \\ \underbrace{\begin{smallmatrix} \mathsf{affine} \\ \mathsf{nonlinearity} \end{smallmatrix}}_{\mathsf{nonlinearity}}$$

Matrix that transforms the embedding from dimension R^d to R^H.

Parameters θ include M and everything in pink.

Define the n-gram probability as follows:



Sum over all n-1 history tokens.

Parameters θ include M and everything in pink.

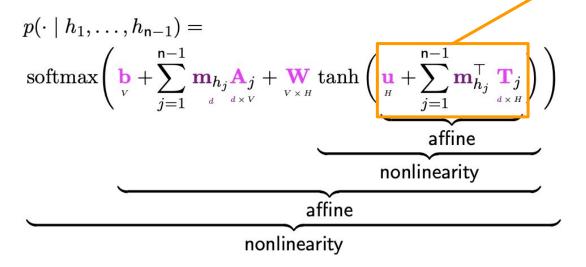
Bias term.

Define the n-gram probability as follows:

$$p(\cdot \mid h_1, \dots, h_{\mathsf{n}-1}) = \\ \operatorname{softmax} \left(\underbrace{\begin{array}{c} \mathbf{b} \\ \mathbf{b} \\ \end{array}}_{v} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j} \underbrace{\begin{array}{c} \mathbf{A}_j \\ \\ \\ \end{array}}_{d \times v} + \underbrace{\begin{array}{c} \mathbf{W} \\ \\ \end{array}}_{v \times H} \tanh \left(\underbrace{\begin{array}{c} \mathbf{u} \\ \\ \\ \end{array}}_{H} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}^{\mathsf{T}} \underbrace{\begin{array}{c} \mathbf{T}_j \\ \\ \\ \end{array}}_{d \times H} \right) \right) \\ \text{affine} \\ \\ \text{nonlinearity} \\ \end{array}$$

Parameters θ include M and everything in pink.

Define the n-gram probability as follows:



Parameters θ include M and everything in pink.

Hyperparameters: dimensionalities d and H

Affine

Define the n-gram probability as follows:

$$p(\cdot \mid h_1, \dots, h_{\mathsf{n}-1}) = \\ \operatorname{softmax} \left(\underbrace{\begin{smallmatrix} \mathbf{b} \\ \mathbf{b} \end{smallmatrix}}_{V} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j} \mathbf{A}_{j} + \underbrace{\bigvee_{V \times H} \mathsf{tanh} \left(\underbrace{\underbrace{\mathbf{u}}_{H} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}^{\top} \mathbf{T}_{j}}_{\mathsf{d} \times H} \right) \right)}_{\mathsf{affine}}$$

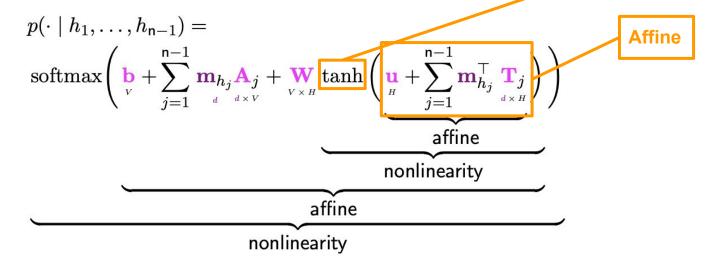
$$\mathsf{nonlinearity}$$

Passing the value through a nonlinearity.

Parameters θ include M and everything in pink.

Define the n-gram probability as follows:

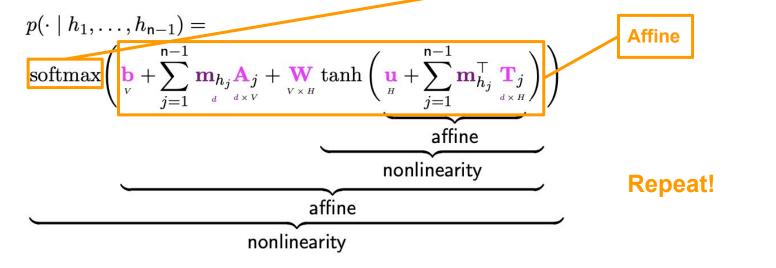
Nonlinearity



Parameters θ include M and everything in pink.

Define the n-gram probability as follows:

Nonlinearity

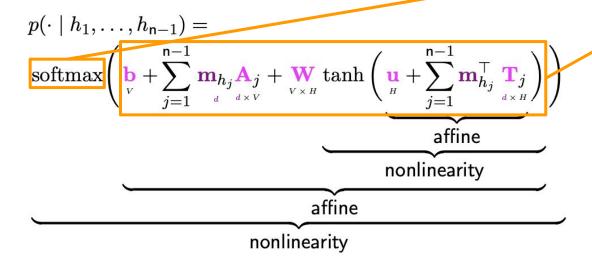


Parameters θ include M and everything in pink.

Define the n-gram probability as follows:

Nonlinearity

Affine



Parameters θ include M and everything in pink.

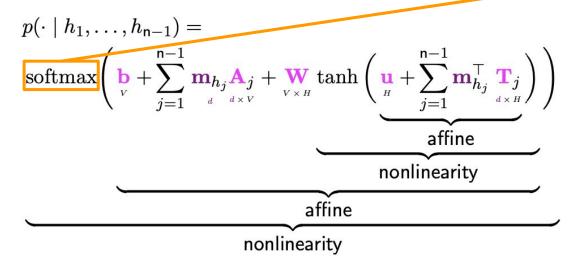
Hyperparameters: dimensionalities d and H

Typical pattern

affine, nonlinear, affine, nonlinear,

. . .

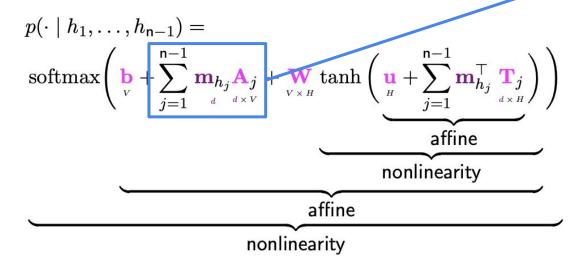
Define the n-gram probability as follows:



Softmax to ensure the output sums to 1.

Parameters θ include M and everything in pink.

Define the n-gram probability as follows:

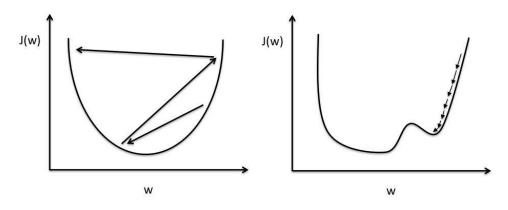


Here we also added the affine transformation of embedding of history tokens too.

Parameters θ include M and everything in pink.

Feedforward Neural Network: Gradient Descent

- Gradient descent is an update on every parameter each iteration
- Does not guarantee to give the optimal solution (gives a local minimum)
- Batch size, epoch, learning rate, various optimizers
 - Stochastic Gradient Descent (SGD), Adam, Adadelta, ...



Large learning rate: Overshooting.

Small learning rate: Many iterations until convergence and trapping in local minima.

Feedforward Neural Network: Hyperparameters

$$D = \underbrace{Vd}_{\mathbf{M}} + \underbrace{V}_{\mathbf{b}} + \underbrace{(\mathbf{n} - 1)dV}_{\mathbf{A}} + \underbrace{VH}_{\mathbf{W}} + \underbrace{H}_{\mathbf{u}} + \underbrace{(\mathbf{n} - 1)dH}_{\mathbf{T}}$$

For Bengio et al. (2003):

- ightharpoonup V pprox 18000 (after OOV processing)
- \rightarrow $d \in \{30, 60\}$
- $ightharpoonup H \in \{50, 100\}$
- ▶ n-1=5

Feedforward Neural Network: Hyperparameters

$$D = \underbrace{Vd}_{\mathbf{M}} + \underbrace{V}_{\mathbf{b}} + \underbrace{(\mathbf{n} - 1)dV}_{\mathbf{A}} + \underbrace{VH}_{\mathbf{W}} + \underbrace{H}_{\mathbf{u}} + \underbrace{(\mathbf{n} - 1)dH}_{\mathbf{T}}$$

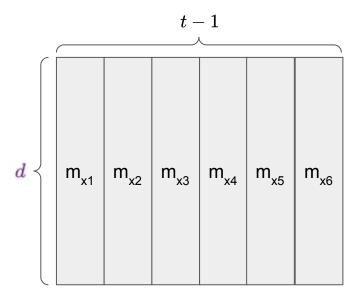
For Bengio et al. (2003):

- ightharpoonup V pprox 18000 (after OOV processing)
- $d \in \{30, 60\}$
- \blacksquare $H \in \{50, 100\}$
- n-1=5

Tune hyperparameters on dev set

Convolutional Neural Network

Start with
$$\mathbf{X}^{(0)} = \left[\mathbf{m}_{x_1}; \mathbf{m}_{x_2}; \dots; \mathbf{m}_{x_{t-1}}\right]$$
.

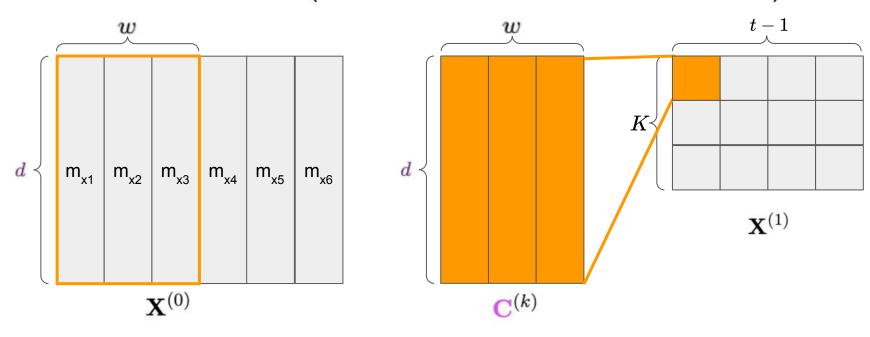


$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$

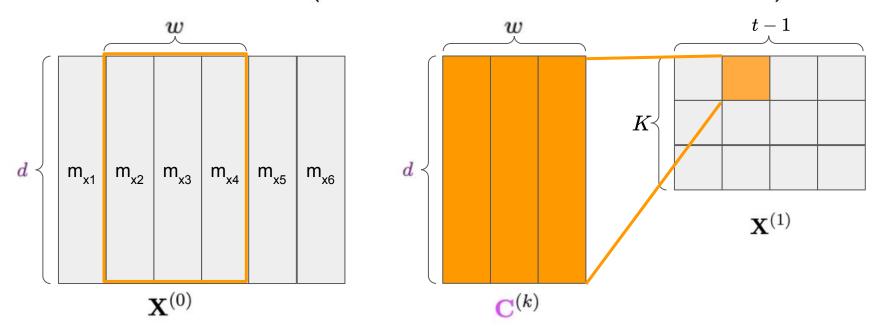
f is a nonlinearity (like tanh). w is the width of the sliding window. Each k is a different "filter" and each m is a word position.

Hyperparameters: number of layers, and, at every layer, f, w, number of filters

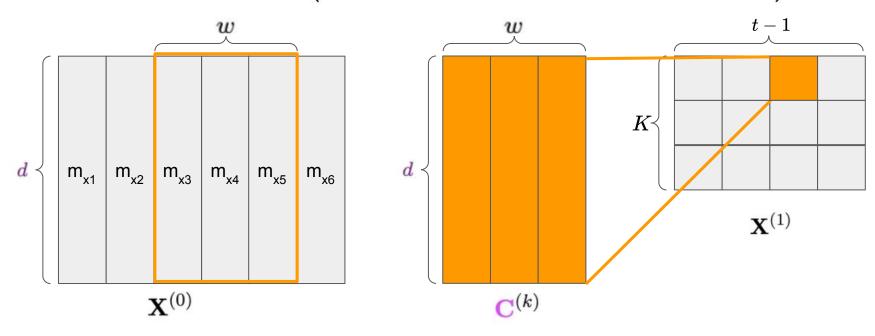
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



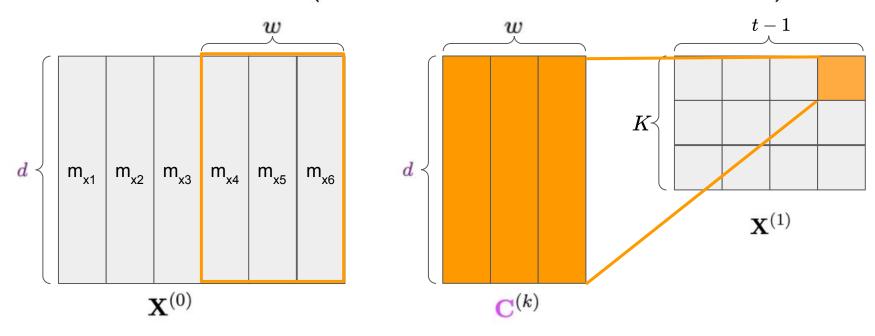
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



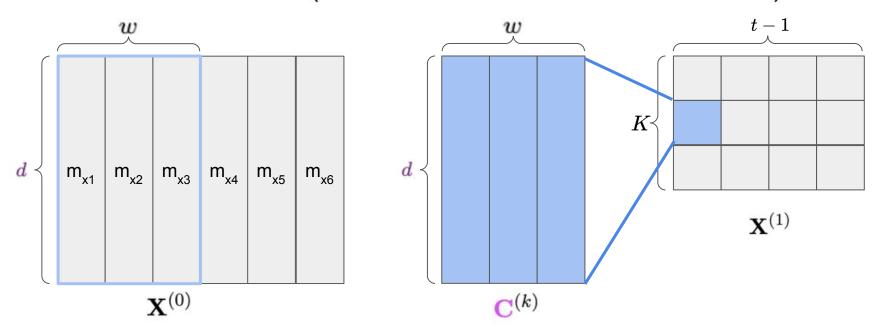
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



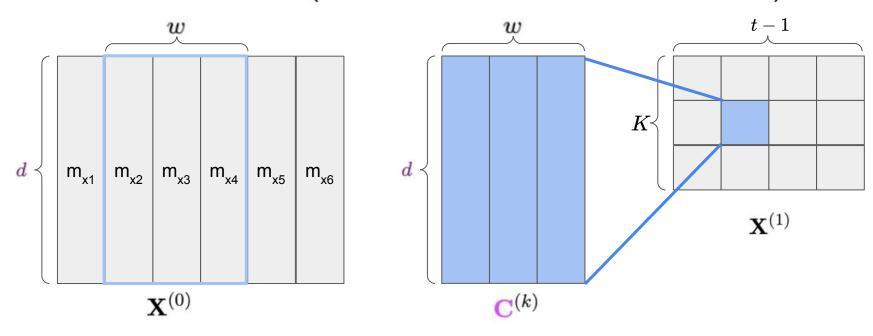
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



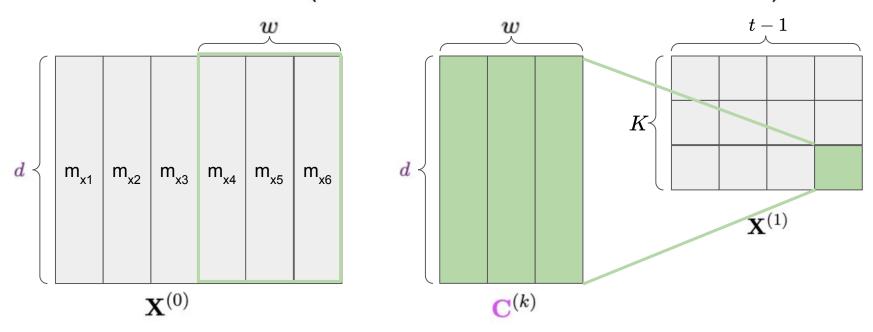
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



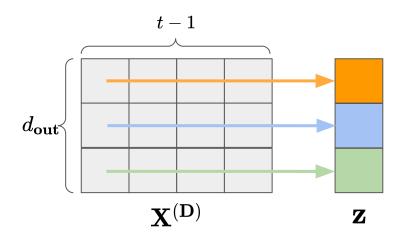
$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



$$\mathbf{X}^{(1)}[k,m] = f\left(\mathbf{b}_k + \sum_{i=1}^d \sum_{j=1}^w \mathbf{C}^{(k)}[i,j] \cdot \mathbf{X}^{(0)}[i,m+j-1]\right)$$



Convolutional Neural Network: Pooling



Pooling takes $\mathbf{X}^{(D)} \in \mathbb{R}^{d_{out} \times (t-1)}$ and maps it into $\mathbb{R}^{d_{out}}$.

Two standard options (with no additional parameters) are max pooling,

$$z_k = \max_j \mathbf{X}^{(D)}[k, j];$$

and average pooling,

$$z_k = rac{1}{t-1} \sum_{j=1}^{t-1} \mathbf{X}^{(D)}[k,j].$$

Finally, softmax(z) gives a probability distribution over outputs.

Q & A