Conditional Random Fields

CSE 447 / 517

February 17, 2022 (Week 7)

Readings: Eisenstein (2019) 7 and 8

Logistics

- A6 is due tomorrow (Friday 2/18 11:59PM)

Agenda

- Conditional Random Field Review
  - Viterbi Algorithm
  - Forward Algorithm
- Quiz 6 Solutions
- Q & A

Task: Sequence Labeling

Problem: Given a sequence, label each element with from a discrete set of labels.

Example: Part-of-Speech tagging

time flies like an arrow

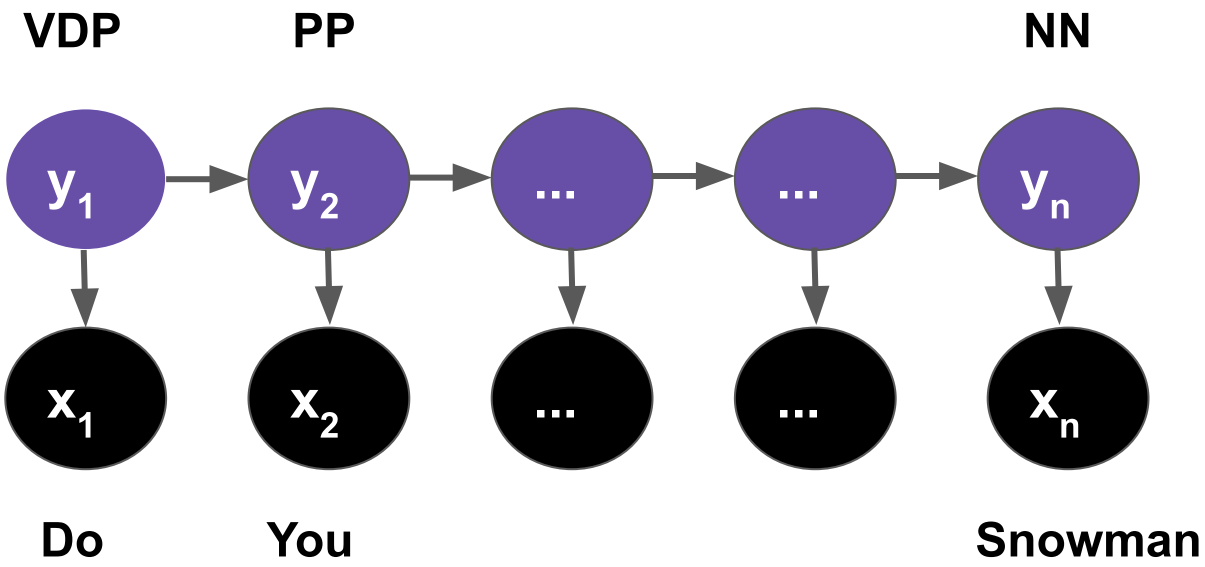
$\Downarrow$

N, V, Prep, Det, N

Notation: $\langle x_1 \rightarrow y_1, x_2 \rightarrow y_2, \ldots, x_n \rightarrow y_n \rangle$, each $y_i \in L$

Conditional Random Field: Motivation

Previously: Hidden Markov Model (HMM)



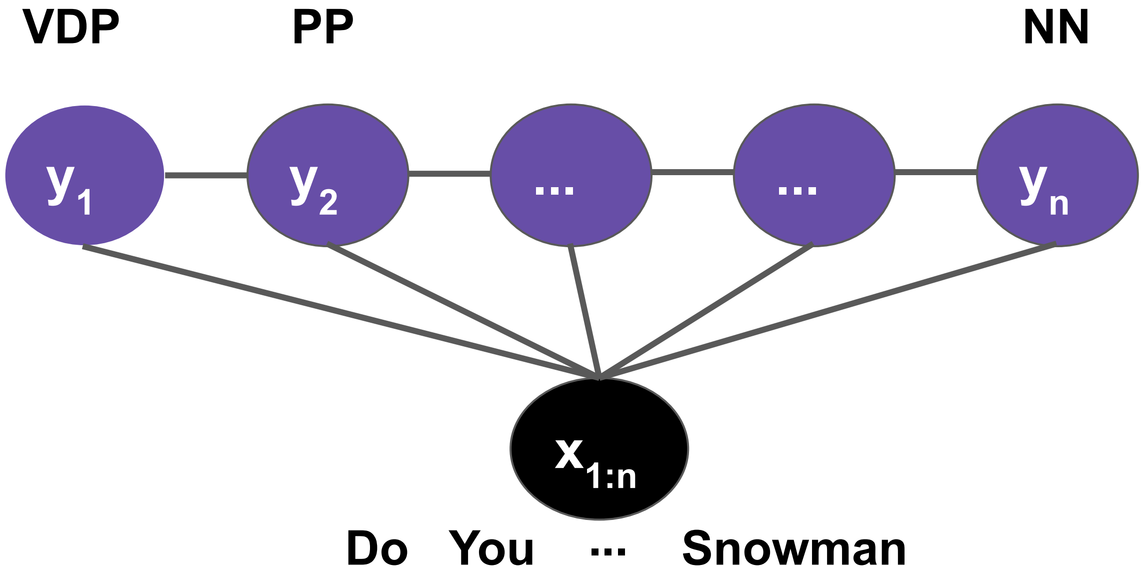
<u>Assumptions:</u>

1. y_i is conditionally independent of prior states given y_{i-1}

2. Observations x_i are conditionally independent of each other and states given y_i

Conditional Random Field: Motivation

In general, CRFs do not require these independence assumptions.



Linear CRFs follow similar structure to HMM

Conditional Random Field: Motivation

HMM (generative model) - models joint distribution

p(x,y)

$$\prod_{i=1}^{N} p(y_i|y_{i-1})p(x_i|y_i)$$

CRF (discriminative model) - directly models conditional distribution

p(y|x)

$$\prod_{i=1}^{N} p(y_i|y_{i-1}, x_i)$$

Conditional Random Field: Overview

Score function:

$$\text{Score}(\mathbf{x},\ \mathbf{y}) = \sum_{i=0}^{n} s(\mathbf{x}, i, y_i, y_{i+1})$$

$s(\mathbf{x}, i, y_i, y_{i+1})$ tells us how good is the label assignments y_i and y_{i+1} are given "access" to the whole input sequence **x** and the position i.

We let y_0 be the start symbol (START) and y_{n+1} be the stop symbol (STOP).

Notation reminder: bold variables are vectors.

Conditional Random Field: Decoding

Decoding: given the input sequence **x** and the score function, what is the best output sequence $\hat{\mathbf{y}}$?

$$\begin{aligned}\hat{\mathbf{y}} &= \text{argmax}_{\mathbf{y}}\ \text{Score}(\mathbf{x},\ \mathbf{y}) \\ &= \text{argmax}_{(y_0, y_1, y_2 \ldots y_{n+1})} \sum_{i=0}^{n} s(\mathbf{x}, i, y_i, y_{i+1})\end{aligned}$$

Note: these decisions are not local!

Conditional Random Field: Decoding

Decoding: given the input sequence **x** and the score function, what is the best output sequence $\hat{\mathbf{y}}$?

$$\hat{\mathbf{y}} = \text{argmax}_{\mathbf{y}}\ \text{Score}(\mathbf{x},\ \mathbf{y})$$
$$= \boxed{\text{argmax}_{(y_0, y_1, y_2 \ldots y_{n+1})}} \sum_{i=0}^{n} s(\mathbf{x}, i, y_i, y_{i+1})$$

This is argmax over all possible sequences $(y_1, y_2, \ldots y_n)$!

Note: these decisions are not local!

Conditional Random Field: Decoding

Decoding: given the input sequence **x** and the score function, what is the best output sequence $\hat{\mathbf{y}}$?

$$\begin{aligned}\hat{\mathbf{y}} &= \operatorname{argmax}_{\mathbf{y}} \operatorname{Score}(\mathbf{x},\, \mathbf{y}) \\ &= \boxed{\operatorname{argmax}_{(y_0, y_1, y_2 \ldots y_{n+1})}} \sum_{i=0}^{n} s(\mathbf{x}, i, y_i, y_{i+1})\end{aligned}$$

This is argmax over all possible sequences $(y_1, y_2, \ldots y_n)$!

Note: these decisions are not local!

Naively: To find $\hat{\mathbf{y}}$, we can iterate over all possible sequences. Given label set size of L and sequence length of n, this is O(L^n) 😢.

Viterbi Algorithm: Overview

Problem: to solve the decoding problem efficiently.

Solution: dynamic programming, specifically, Viterbi algorithm.

Intuition: the best label sequence that end in (y_{i-1},y_i) (i.e. y_1,y_2,y_3,...y_{i-1}, y_i) has to have the best prefix (y_1,y_2,y_3,...y_{i-1}).

Viterbi Algorithm: Recurrence

Let $\heartsuit_i(y)$ be the score of the best label sequence for $(x_1, x_2 \ldots x_i)$ that ends in y.

Define it by recurrence:

$$\heartsuit_i(y) = \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}$$

Base case: the best possible label sequence for (x_1) that ends in y.

$$\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$$

Viterbi Algorithm: Recurrence

Let $\heartsuit_i(y)$ be the score of the best label sequence for $(x_1, x_2 \ldots x_i)$ that ends in y.

Define it by recurrence:

$$\heartsuit_i(y) = \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}$$

Try every possible label for y_{i-1}.

Base case: the best possible label sequence for (x_1) that ends in y.

$$\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$$

Viterbi Algorithm: Recurrence

Let $\heartsuit_i(y)$ be the score of the best label sequence for $(x_1, x_2 \ldots x_i)$ that ends in y.

Define it by recurrence:

$$\heartsuit_i(y) = \boxed{\max_{y_{i-1} \in \mathcal{L}}} \boxed{s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}}$$

Try every possible label for y_{i-1}.

What would the best score for sequence ending in (y_{i-1}, y_i) be?

Base case: the best possible label sequence for (x_1) that ends in y.

$$\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
L	ℓ_1	$\heartsuit_1(\ell_1)$			
	ℓ_2				
	ℓ_3				

$$\heartsuit_1(l_1) = s(\mathbf{x}, 0, \mathrm{START}, l_1)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

Label set

L

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$			
ℓ_2				
ℓ_3				

$$\heartsuit_1(l_1) = s(\mathbf{x}, 0, \mathrm{START}, l_1)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

Input sequence with 4 tokens

Label set

L

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$			
ℓ_2				
ℓ_3				

$$\heartsuit_1(l_1) = s(\mathbf{x}, 0, \mathrm{START}, l_1)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

Input sequence with 4 tokens

Label set

L

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$			
ℓ_2				
ℓ_3				

The score of the best label sequence for (x_1) that ends in ℓ_1

$$\boxed{\heartsuit_1(l_1) = s(\mathbf{x}, 0, \mathrm{START}, l_1)}$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
L	ℓ_1	$\heartsuit_1(\ell_1)$			
	ℓ_2	$\heartsuit_1(\ell_2)$			
	ℓ_3	$\heartsuit_1(\ell_3)$			

$$\heartsuit_1(l_1) = s(\mathbf{x}, 0, \mathrm{START}, l_1)$$

...

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
L	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$		
	ℓ_2	$\heartsuit_1(\ell_2)$			
	ℓ_3	$\heartsuit_1(\ell_3)$			

$$\heartsuit_2(l_1) = \max_{y \in L} s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
	ℓ_1	$\heartsuit_1(\ell_1)$	$\boxed{\heartsuit_2(\ell_1)}$		
L	ℓ_2	$\heartsuit_1(\ell_2)$			
	ℓ_3	$\heartsuit_1(\ell_3)$			

The score of the best label sequence for (x_1,x_2) that ends in ℓ_1

$$\heartsuit_2(l_1) = \max_{y \in L} s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$	$\boxed{\heartsuit_2(\ell_1)}$		
ℓ_2	$\heartsuit_1(\ell_2)$			
ℓ_3	$\heartsuit_1(\ell_3)$			

L

The score of the best label sequence for (x_1,x_2) that ends in ℓ_1

Iterate over all possible labels

$$\heartsuit_2(l_1) = \boxed{\max_{y \in L}} s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
L	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$		
	ℓ_2	$\heartsuit_1(\ell_2)$			
	ℓ_3	$\heartsuit_1(\ell_3)$			

The score of the best label sequence for (x_1,x_2) that ends in ℓ_1

Iterate over all possible labels

$$\heartsuit_2(l_1) = \boxed{\max_{y \in L}} \boxed{s(\mathbf{x}, 1, y, l_1)} + \heartsuit_1(y)$$

How good is the label pair y and ℓ_1 at position 1 and 2?

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$	$\boxed{\heartsuit_2(\ell_1)}$		
ℓ_2	$\heartsuit_1(\ell_2)$			
ℓ_3	$\heartsuit_1(\ell_3)$			

L

The score of the best label sequence for (x_1,x_2) that ends in ℓ_1

Iterate over all possible labels

$$\heartsuit_2(l_1) = \boxed{\max_{y \in L}} \boxed{s(\mathbf{x}, 1, y, l_1)} + \boxed{\heartsuit_1(y)}$$

How good is the label pair y and ℓ_1 at position 1 and 2?

How good is the prefix that ends in y?

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4
ℓ_1	$\boxed{\heartsuit_1(\ell_1)}$	$\boxed{\heartsuit_2(\ell_1)}$		
ℓ_2	$\boxed{\heartsuit_1(\ell_2)}$			
ℓ_3	$\boxed{\heartsuit_1(\ell_3)}$			

L

$$\heartsuit_2(l_1) = \max_{y \in L} \boxed{s(\mathbf{x}, 1, y, l_1)} + \boxed{\heartsuit_1(y)}$$

Good news! We have both of these!

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

		x_1	x_2	x_3	x_4
L	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $\boxed{bp_2(\ell_1)}$		
	ℓ_2	$\heartsuit_1(\ell_2)$			
	ℓ_3	$\heartsuit_1(\ell_3)$			

$$\heartsuit_2(l_1) = \max_{y \in L} s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)$$

$$\boxed{\mathrm{b}p_2(l_1) = \mathrm{argmax}_{y \in L}\, s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)}$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $\boxed{bp_2(\ell_1)}$		
ℓ_2	$\heartsuit_1(\ell_2)$			
ℓ_3	$\heartsuit_1(\ell_3)$			

$$\heartsuit_2(l_1) = \max_{y \in L} s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)$$

$$\boxed{\mathrm{b}p_2(l_1) = \mathrm{argmax}_{y \in L}\, s(\mathbf{x}, 1, y, l_1) + \heartsuit_1(y)}$$

Just keep track of which label y gave us the best score!

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $bp_2(\ell_1)$	$\heartsuit_3(\ell_1)$ $bp_3(\ell_1)$	$\heartsuit_4(\ell_1)$ $bp_4(\ell_1)$
ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $bp_2(\ell_2)$	$\heartsuit_3(\ell_2)$ $bp_3(\ell_2)$	$\heartsuit_4(\ell_2)$ $bp_4(\ell_2)$
ℓ_3	$\heartsuit_1(\ell_3)$	$\heartsuit_2(\ell_3)$ $bp_2(\ell_3)$	$\heartsuit_3(\ell_3)$ $bp_3(\ell_3)$	$\heartsuit_4(\ell_3)$ $bp_4(\ell_3)$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4	
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $bp_2(\ell_1)$	$\heartsuit_3(\ell_1)$ $bp_3(\ell_1)$	$\heartsuit_4(\ell_1)$ $bp_4(\ell_1)$	
ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $bp_2(\ell_2)$	$\heartsuit_3(\ell_2)$ $bp_3(\ell_2)$	$\heartsuit_4(\ell_2)$ $bp_4(\ell_2)$	$\boxed{\heartsuit_5(\text{STOP})}$ $bp_5(\text{STOP})$
ℓ_3	$\heartsuit_1(\ell_3)$	$\heartsuit_2(\ell_3)$ $bp_2(\ell_3)$	$\heartsuit_3(\ell_3)$ $bp_3(\ell_3)$	$\heartsuit_4(\ell_3)$ $bp_4(\ell_3)$	
STOP	\	\	\	\	

$$\heartsuit_5(\text{STOP}) = \max_{y \in L} s(\mathbf{x}, 4, y, \text{STOP}) + \heartsuit_4(y)$$
$$= \max_{\mathbf{y}} \text{Score}(\mathbf{x},\ \mathbf{y})$$

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4	
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $bp_2(\ell_1)$	$\heartsuit_3(\ell_1)$ $bp_3(\ell_1)$	$\heartsuit_4(\ell_1)$ $bp_4(\ell_1)$	
ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $bp_2(\ell_2)$	$\heartsuit_3(\ell_2)$ $bp_3(\ell_2)$	$\heartsuit_4(\ell_2)$ $bp_4(\ell_2)$	$\boxed{\heartsuit_5(\text{STOP})}$ $bp_5(\text{STOP})$
ℓ_3	$\heartsuit_1(\ell_3)$	$\heartsuit_2(\ell_3)$ $bp_2(\ell_3)$	$\heartsuit_3(\ell_3)$ $bp_3(\ell_3)$	$\heartsuit_4(\ell_3)$ $bp_4(\ell_3)$	
STOP	\	\	\	\	

$$\heartsuit_5(\text{STOP}) = \max_{y \in L} s(\mathbf{x}, 4, y, \text{STOP}) + \heartsuit_4(y)$$
$$= \boxed{\max_{\mathbf{y}} \text{Score}(\mathbf{x},\ \mathbf{y})}$$

We found the max possible score!

Viterbi Algorithm in Action

Fill out this table from left to right, and backtrack from right to left.

	x_1	x_2	x_3	x_4	
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $bp_2(\ell_1)$	$\heartsuit_3(\ell_1)$ $bp_3(\ell_1)$	$\heartsuit_4(\ell_1)$ $bp_4(\ell_1)$	
ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $bp_2(\ell_2)$	$\heartsuit_3(\ell_2)$ $bp_3(\ell_2)$	$\heartsuit_4(\ell_2)$ $bp_4(\ell_2)$	$\heartsuit_5(\text{STOP})$ $\boxed{bp_5(\text{STOP})}$
ℓ_3	$\heartsuit_1(\ell_3)$	$\heartsuit_2(\ell_3)$ $bp_2(\ell_3)$	$\heartsuit_3(\ell_3)$ $bp_3(\ell_3)$	$\heartsuit_4(\ell_3)$ $bp_4(\ell_3)$	
STOP	\	\	\	\	

Follow the back pointers to decode!

Viterbi Algorithm: Performance

Assume s(**x**, i, y_{i-1}, y_i) is constant time and space.

- Space: O(nL)
  - All we need in this case is to fill in the data structure.
  - It is a table with O(n) columns and O(L) rows.
- Runtime: O(nL^2)
  - O(L): For each cell, we need to find max / argmax over label set *L*.
  - There are O(nL) cells.

n

L

<table>
<tr><td></td><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td><td></td></tr>
<tr><td>ℓ_1</td><td>$\heartsuit_1(\ell_1)$</td><td>$\heartsuit_2(\ell_1)$
$bp_2(\ell_1)$</td><td>$\heartsuit_3(\ell_1)$
$bp_3(\ell_1)$</td><td>$\heartsuit_4(\ell_1)$
$bp_4(\ell_1)$</td><td rowspan="4">$\heartsuit_5$(STOP)
bp_5(STOP)</td></tr>
<tr><td>ℓ_2</td><td>$\heartsuit_1(\ell_2)$</td><td>$\heartsuit_2(\ell_2)$
$bp_2(\ell_2)$</td><td>$\heartsuit_3(\ell_2)$
$bp_3(\ell_2)$</td><td>$\heartsuit_4(\ell_2)$
$bp_4(\ell_2)$</td></tr>
<tr><td>ℓ_3</td><td>$\heartsuit_1(\ell_3)$</td><td>$\heartsuit_2(\ell_3)$
$bp_2(\ell_3)$</td><td>$\heartsuit_3(\ell_3)$
$bp_3(\ell_3)$</td><td>$\heartsuit_4(\ell_3)$
$bp_4(\ell_3)$</td></tr>
<tr><td>STOP</td><td>\</td><td>\</td><td>\</td><td>\</td></tr>
</table>

Remark: s(**x**, i, y_{i-1}, y_i) is often not constant time and space.

Conditional Random Field: Decoding

Decoding: given the input sequence **x** and the score function, what is the best output sequence $\hat{\mathbf{y}}$?

$$
\begin{aligned}
\hat{\mathbf{y}} &= \mathrm{argmax}_{\mathbf{y}}\ \mathrm{Score}(\mathbf{x},\ \mathbf{y}) \\
&= \mathrm{argmax}_{(y_0, y_1, y_2 \dots y_{n+1})} \sum_{i=0}^{n} s(\mathbf{x}, i, y_i, y_{i+1})
\end{aligned}
$$

Just use Viterbi!

Runtime: Viterbi $O(nL^2)$ vs Naive $O(n^L)$ 🤯🤯🤯

Conditional Random Field: Learning

Training: Given input sequences **x** and gold output sequences **y**, what is the best θ* such that we maximize P(**y** | **x** ; θ) over all observations?

$$Z(\boldsymbol{x};\boldsymbol{\theta}) = \sum_{\boldsymbol{y}' \in \mathcal{Y}(\boldsymbol{x})} \exp \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}'; \boldsymbol{\theta})$$

$$p_{\mathrm{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta}) = \frac{\exp \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta})}{Z(\boldsymbol{x};\boldsymbol{\theta})}$$

$$-\log p_{\mathrm{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta}) = -\mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta}) + \log Z(\boldsymbol{x};\boldsymbol{\theta})$$

Forward Algorithm: Overview

Problem: We need to compute the partition function Z(x;θ) in order to train our CRF model.

$$Z(\boldsymbol{x};\boldsymbol{\theta}) = \sum_{\boldsymbol{y}' \in \mathcal{Y}(\boldsymbol{x})} \exp \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}'; \boldsymbol{\theta})$$

Solution: Dynamic programming similar to Viterbi algorithm.

Recurrence:

$$\alpha_i(y_i) = \sum_{y_{i-1} \in L} \exp(s(\mathbf{x}, i-1, y_{i-1}, y)) \times \alpha_{i-1}(y_{i-1})$$

Forward Algorithm in Action

Input sequence with 4 tokens

Label set

	x_1	x_2	x_3	x_4
ℓ_1	$\alpha_1(y = \ell_1)$			
ℓ_2				
ℓ_3				

The exponentiated score of the label sequence for (x_1) that ends in ℓ_1

$$\alpha_1(l_1) = e^{s(\mathbf{x},0,\mathrm{START},l_1)}$$

Forward Algorithm in Action

	x_1	x_2	x_3	x_4
ℓ_1	$\alpha_1(\ell_1)$	$\alpha_2(\ell_1)$		
ℓ_2	$\alpha_1(\ell_2)$			
ℓ_3	$\alpha_1(\ell_3)$			

The sum of label sequence scores for (x_1, x_2) that end in ℓ_1

Iterate over all possible prefixes

$$\alpha_2(l_1) = \boxed{\sum_{y_1 \in L}} e^{s(\mathbf{x},1,y_1,l_1)} \boxed{\times \alpha_1(y_1)}$$

Scores are exponentiated, so we multiply

Forward Algorithm in Action

	x_1	x_2	x_3	x_4
ℓ_1	$\alpha_1(\ell_1)$	$\alpha_2(\ell_1)$	$\alpha_3(\ell_1)$	$\alpha_4(\ell_1)$
ℓ_2	$\alpha_1(\ell_2)$	$\alpha_2(\ell_2)$	$\alpha_3(\ell_2)$	$\alpha_4(\ell_2)$
ℓ_3	$\alpha_1(\ell_3)$	$\alpha_2(\ell_3)$	$\alpha_3(\ell_3)$	$\alpha_4(\ell_3)$

Forward Algorithm in Action

	x_1	x_2	x_3	x_4	
ℓ_1	$\alpha_1(\ell_1)$	$\alpha_2(\ell_1)$	$\alpha_3(\ell_1)$	$\alpha_4(\ell_1)$	
ℓ_2	$\alpha_1(\ell_2)$	$\alpha_2(\ell_2)$	$\alpha_3(\ell_2)$	$\alpha_4(\ell_2)$	$\alpha_5(\text{STOP})$
ℓ_3	$\alpha_1(\ell_3)$	$\alpha_2(\ell_3)$	$\alpha_3(\ell_3)$	$\alpha_4(\ell_3)$	
STOP	\	\	\	\	

Partition function $Z(\mathbf{x}; \theta)$

Forward Algorithm: Performance

Assume s($\mathbf{x}$, i, y_{i-1}, y_i) is constant time and space.

- Space: $O(nL)$
- Runtime: $O(nL^2)$

Computation at each step is slightly different from Viterbi, but matches complexity

Viterbi

n

L

	x_1	x_2	x_3	x_4	
ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $bp_2(\ell_1)$	$\heartsuit_3(\ell_1)$ $bp_3(\ell_1)$	$\heartsuit_4(\ell_1)$ $bp_4(\ell_1)$	
ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $bp_2(\ell_2)$	$\heartsuit_3(\ell_2)$ $bp_3(\ell_2)$	$\heartsuit_4(\ell_2)$ $bp_4(\ell_2)$	$\heartsuit_5(\text{STOP})$ $bp_5(\text{STOP})$
ℓ_3	$\heartsuit_1(\ell_3)$	$\heartsuit_2(\ell_3)$ $bp_2(\ell_3)$	$\heartsuit_3(\ell_3)$ $bp_3(\ell_3)$	$\heartsuit_4(\ell_3)$ $bp_4(\ell_3)$	
STOP	\	\	\	\	

n

Forward

L

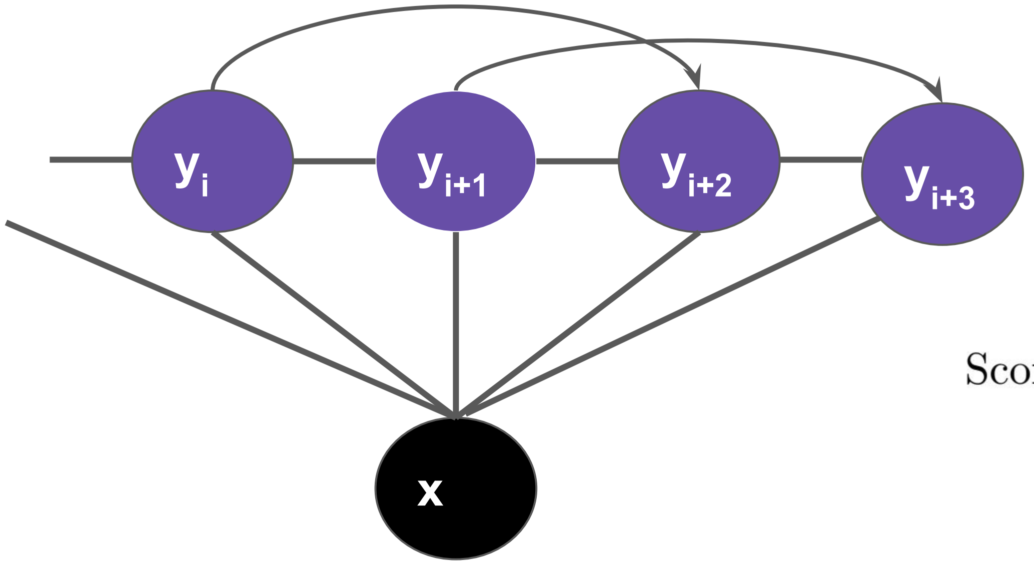
	x_1	x_2	x_3	x_4	
ℓ_1	$\alpha_1(\ell_1)$	$\alpha_2(\ell_1)$	$\alpha_3(\ell_1)$	$\alpha_4(\ell_1)$	
ℓ_2	$\alpha_1(\ell_2)$	$\alpha_2(\ell_2)$	$\alpha_3(\ell_2)$	$\alpha_4(\ell_2)$	$\alpha_5(\text{STOP})$
ℓ_3	$\alpha_1(\ell_3)$	$\alpha_2(\ell_3)$	$\alpha_3(\ell_3)$	$\alpha_4(\ell_3)$	
STOP	\	\	\	\	

Conditional Random Field: Learning

$$-\log p_{\mathrm{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta}) = -\operatorname{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta}) + \log Z(\boldsymbol{x}; \boldsymbol{\theta})$$

Use stochastic gradient descent to minimize log loss

Conditional Random Field: Beyond 1st Order



Second-Order CRF Model

General Form:

$$\text{Score}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=0}^{n} s(\boldsymbol{x}, i, \boldsymbol{y}_{i:i+m})$$

Quiz 6 - Problem 1

You are about to run the Viterbi algorithm using a label set of size 10, on a sequence of length 9; your model is an HMM. The HMM is smoothed, so that p(x | y) > 0 for every vocabulary word x and every label y, and p(y' | y) > 0 for every pair of labels y and y'. How many possible full-sequence labelings $\langle y_0 = \text{start}, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} = \text{stop}\rangle$ are there?

Note that the start and stop labels are given! Clarification: the 10 labels allowed for regular non-stop-symbol words do not include the start and stop labels.

Quiz 6 - Problem 1

You are about to run the Viterbi algorithm using a label set of size 10, on a sequence of length 9; your model is an HMM. The HMM is smoothed, so that $p(x \mid y) > 0$ for every vocabulary word x and every label y, and $p(y' \mid y) > 0$ for every pair of labels y and y'. How many possible full-sequence labelings $\langle y_0 = \text{start}, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} = \text{stop}\rangle$ are there?

Note that the start and stop labels are given! Clarification: the 10 labels allowed for regular non-stop-symbol words do not include the start and stop labels.

9 positions, 10 options for each position -> 10^9 = **1,000,000,000**

Quiz 6 - Problem 2

We know that "time flies like an arrow; fruit flies like a banana". Now let us label each word in the sequence "**fruit flies like bananas**". For simplicity, we consider three labels **L = {N, V, O}**. We apply Viterbi algorithm to decode the sentence. Compute the values for each blank space and record the back pointer.

Let the score function $\mathbf{s(x,i,y_i,y_{i-1}) = \log(e(x_i|y_i) * q(y_i|y_{i-1}))}$*, where e is the emission probability and q is the transition probability (given on the next slide).

We have the following recurrence: $\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1})\right) + \heartsuit_{i-1}(y_{i-1})$

**Note: use natural log!*

q(N|START)=0.6	q(N|N)=0.4	q(N|V)=0.5	q(N|O)=0.7
q(V|START)=0.3	q(V|N)=0.3	q(V|V)=0.1	q(V|O)=0.1
q(O|START)=0.1	q(O|N)=0.1	q(O|V)=0.2	q(O|O)=0.1
q(STOP|START)=0.0	q(STOP|N)=0.2	q(STOP|V)=0.2	q(STOP|O)=0.1
e(*|STOP) = 1	e(fruit|N)=0.3	e(fruit|V)=0.1	e(fruit|O)=0.1
	e(flies|N)=0.3	e(flies|V)=0.5	e(flies|O)=0.0
	e(like|N)=0.1	e(like|V)=0.4	e(like|O)=0.3
	e(bananas|N)=0.3	e(bananas|V)=0.0	e(bananas|O)=0.6

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log \left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1}) \right) + \heartsuit_{i-1}(y_{i-1})$$

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)= bp_1 (N)=	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1})\right) + \heartsuit_{i-1}(y_{i-1})$$

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)= bp_1 (N)=	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(N) = \max($
$\log(e(\text{“fruit”} \mid N) * q(N \mid START)) + \heartsuit_0(START)$
$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=$ $bp_1(N)=$	$\heartsuit_2(N)=$ $bp_2(N) =$	$\heartsuit_3(N) =$ $bp_3(N) =$	$\heartsuit_4(N)=$ $bp_4(N) =$	
$\heartsuit_0(START)=0$	$\heartsuit_1(V)=$ $bp_1(V)=$	$\heartsuit_2(V)=$ $bp_2(V) =$	$\heartsuit_3(V)=$ $bp_3(V) =$	$\heartsuit_4(V)=$ $bp_4(V) =$	$\heartsuit_5(STOP)=$ $bp_5(STOP) =$
	$\heartsuit_1(O)=$ $bp_1(O)=$	$\heartsuit_2(O)=$ $bp_2(O) =$	$\heartsuit_3(O)=$ $bp_3(O) =$	$\heartsuit_4(O)=$ $bp_4(O) =$	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(N) = max(

log(e("fruit" | N) * q(N | START)) + $\heartsuit_0$(START)

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)= bp_1 (N)=	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(N) = \max($

log(**e("fruit" | N)** * **q(N | START)**) + **0**

$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=$ $bp_1(N)=$	$\heartsuit_2(N)=$ $bp_2(N) =$	$\heartsuit_3(N) =$ $bp_3(N) =$	$\heartsuit_4(N)=$ $bp_4(N) =$	
$\heartsuit_0(START)=0$	$\heartsuit_1(V)=$ $bp_1(V)=$	$\heartsuit_2(V)=$ $bp_2(V) =$	$\heartsuit_3(V)=$ $bp_3(V) =$	$\heartsuit_4(V)=$ $bp_4(V) =$	$\heartsuit_5(STOP)=$ $bp_5(STOP) =$
	$\heartsuit_1(O)=$ $bp_1(O)=$	$\heartsuit_2(O)=$ $bp_2(O) =$	$\heartsuit_3(O)=$ $bp_3(O) =$	$\heartsuit_4(O)=$ $bp_4(O) =$	

q(N|START)=0.6	q(N|N)=0.4	q(N|V)=0.5	q(N|O)=0.7
q(V|START)=0.3	q(V|N)=0.3	q(V|V)=0.1	q(V|O)=0.1
q(O|START)=0.1	q(O|N)=0.1	q(O|V)=0.2	q(O|O)=0.1
q(STOP|START)=0.0	q(STOP|N)=0.2	q(STOP|V)=0.2	q(STOP|O)=0.1
e(*|STOP) = 1	**e(fruit|N)=0.3**	e(fruit|V)=0.1	e(fruit|O)=0.1
	e(flies|N)=0.3	e(flies|V)=0.5	e(flies|O)=0.0
	e(like|N)=0.1	e(like|V)=0.4	e(like|O)=0.3
	e(bananas|N)=0.3	e(bananas|V)=0.0	e(bananas|O)=0.6

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(N) = max(
log(0.3 * 0.6) + 0
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)= bp_1 (N)=	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(N) = max(
log(**0.18**)
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)= bp_1 (N)=	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(N)$ = max(
-1.715
)

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$= $bp_1(N)$=	$\heartsuit_2(N)$= $bp_2(N)$ =	$\heartsuit_3(N)$ = $bp_3(N)$ =	$\heartsuit_4(N)$= $bp_4(N)$ =	
$\heartsuit_0(START)$=0	$\heartsuit_1(V)$= $bp_1(V)$=	$\heartsuit_2(V)$= $bp_2(V)$ =	$\heartsuit_3(V)$= $bp_3(V)$ =	$\heartsuit_4(V)$= $bp_4(V)$ =	$\heartsuit_5(STOP)$= $bp_5(STOP)$ =
	$\heartsuit_1(O)$= $bp_1(O)$=	$\heartsuit_2(O)$= $bp_2(O)$ =	$\heartsuit_3(O)$= $bp_3(O)$ =	$\heartsuit_4(O)$= $bp_4(O)$ =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log \left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})} \right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(N) = -1.715$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=	$\heartsuit_2(N)$= bp_2 (N) =	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$= bp_1 (V)=	$\heartsuit_2(V)$= bp_2 (V) =	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$= bp_1 (O)=	$\heartsuit_2(O)$= bp_2 (O) =	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1})\right) + \heartsuit_{i-1}(y_{i-1})$$

The argmax was START.
Note: bp_1 is always START.

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(V) = max(

log(**e("fruit" | V)** * **q(V | START)**) + **$\heartsuit_0$(START)**

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(V) = max(
log(e("fruit" | V) * q(V | START)) + 0
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

q(N|START)=0.6	q(N|N)=0.4	q(N|V)=0.5	q(N|O)=0.7
q(V|START)=0.3	q(V|N)=0.3	q(V|V)=0.1	q(V|O)=0.1
q(O|START)=0.1	q(O|N)=0.1	q(O|V)=0.2	q(O|O)=0.1
q(STOP|START)=0.0	q(STOP|N)=0.2	q(STOP|V)=0.2	q(STOP|O)=0.1
e(*|STOP) = 1	e(fruit|N)=0.3	**e(fruit|V)=0.1**	e(fruit|O)=0.1
	e(flies|N)=0.3	e(flies|V)=0.5	e(flies|O)=0.0
	e(like|N)=0.1	e(like|V)=0.4	e(like|O)=0.3
	e(bananas|N)=0.3	e(bananas|V)=0.0	e(bananas|O)=0.6

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(V) = max(
log(0.1 * 0.3) + 0
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)= bp_1 (V)=	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(V) = -3.507$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=-1.715$ $bp_1(N)$=START	$\heartsuit_2(N)=$ $bp_2(N)=$	$\heartsuit_3(N)=$ $bp_3(N)=$	$\heartsuit_4(N)=$ $bp_4(N)=$	
$\heartsuit_0(START)=0$	$\boxed{\heartsuit_1(V)=-3.507}$ $bp_1(V)=$	$\heartsuit_2(V)=$ $bp_2(V)=$	$\heartsuit_3(V)=$ $bp_3(V)=$	$\heartsuit_4(V)=$ $bp_4(V)=$	$\heartsuit_5(STOP)=$ $bp_5(STOP)=$
	$\heartsuit_1(O)=$ $bp_1(O)=$	$\heartsuit_2(O)=$ $bp_2(O)=$	$\heartsuit_3(O)=$ $bp_3(O)=$	$\heartsuit_4(O)=$ $bp_4(O)=$	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

The argmax was START.
Note: bp_1 is always START.

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\boxed{\heartsuit_1(V)=-3.507}$ bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log \left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})} \right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(O) = max(
log(??? * ???) + ???
)

What would these be?

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1$(O) = max(
log(**e("fruit" | O)** * **q(O | START)**) + **$\heartsuit_0$(START)**
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)= bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(O) = \max($
$\quad \log(0.1 * 0.1) + 0$
$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=-1.715$ $bp_1(N)=START$	$\heartsuit_2(N)=$ $bp_2(N) =$	$\heartsuit_3(N) =$ $bp_3(N) =$	$\heartsuit_4(N)=$ $bp_4(N) =$	
$\heartsuit_0(START)=0$	$\heartsuit_1(V)=-3.507$ $bp_1(V)=START$	$\heartsuit_2(V)=$ $bp_2(V) =$	$\heartsuit_3(V)=$ $bp_3(V) =$	$\heartsuit_4(V)=$ $bp_4(V) =$	$\heartsuit_5(STOP)=$ $bp_5(STOP) =$
	$\heartsuit_1(O)=$ $bp_1(O)=$	$\heartsuit_2(O)=$ $bp_2(O) =$	$\heartsuit_3(O)=$ $bp_3(O) =$	$\heartsuit_4(O)=$ $bp_4(O) =$	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_1(O) = -4.605$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=-1.715$ $bp_1(N)=START$	$\heartsuit_2(N)=$ $bp_2(N) =$	$\heartsuit_3(N) =$ $bp_3(N) =$	$\heartsuit_4(N)=$ $bp_4(N) =$	
$\heartsuit_0(START)=0$	$\heartsuit_1(V)=-3.507$ $bp_1(V)=START$	$\heartsuit_2(V)=$ $bp_2(V) =$	$\heartsuit_3(V)=$ $bp_3(V) =$	$\heartsuit_4(V)=$ $bp_4(V) =$	$\heartsuit_5(STOP)=$ $bp_5(STOP) =$
	$\heartsuit_1(O)=-4.605$ $bp_1(O)=$	$\heartsuit_2(O)=$ $bp_2(O) =$	$\heartsuit_3(O)=$ $bp_3(O) =$	$\heartsuit_4(O)=$ $bp_4(O) =$	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

The argmax was **???**

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

The argmax was START.
Note: bp_1 is always START.

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\boxed{\heartsuit_1\text{(O)=-4.605}}$ bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

$\heartsuit_2$(N) = max(
log(e(“flies” | N) * q(N | N)) + $\heartsuit_1$(N),

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N $\rightarrow$

y_{i-1}=V $\rightarrow$

$\heartsuit_2$(N) = max(
log(e("flies" | N) * q(N | N)) + $\heartsuit_1$(N),
log(e("flies" | N) * q(N | V)) + $\heartsuit_1$(V),
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(N) = max(
log(e("flies" | N) * q(N | N)) + $\heartsuit_1$(N),
log(e("flies" | N) * q(N | V)) + $\heartsuit_1$(V),
log(e("flies" | N) * q(N | O)) + $\heartsuit_1$(O)
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(N) = max(
log(0.3 * 0.4) + -1.715,
log(0.3 * 0.5) + -3.507,
log(0.3 * 0.7) + -4.605
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)= bp_2 (N) =	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underbrace{e(x_i \mid y_i)} \times \underbrace{q(y_i \mid y_{i-1})}\right) + \underbrace{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(N) = \max($

$y_{i-1}=N \rightarrow$ **-3.835**,

$y_{i-1}=V \rightarrow$ -5.404,

$y_{i-1}=O \rightarrow$ -6.165

$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=-1.715$ $bp_1(N)=$START	$\heartsuit_2(N)=$ $bp_2(N)=$	$\heartsuit_3(N)=$ $bp_3(N)=$	$\heartsuit_4(N)=$ $bp_4(N)=$	
$\heartsuit_0(\text{START})=0$	$\heartsuit_1(V)=-3.507$ $bp_1(V)=$START	$\heartsuit_2(V)=$ $bp_2(V)=$	$\heartsuit_3(V)=$ $bp_3(V)=$	$\heartsuit_4(V)=$ $bp_4(V)=$	$\heartsuit_5(\text{STOP})=$ $bp_5(\text{STOP})=$
	$\heartsuit_1(O)=-4.605$ $bp_1(O)=$START	$\heartsuit_2(O)=$ $bp_2(O)=$	$\heartsuit_3(O)=$ $bp_3(O)=$	$\heartsuit_4(O)=$ $bp_4(O)=$	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(N) = -3.835$
$bp_2(N) = N$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=START	$\heartsuit_2(N)$=-3.835 bp_2 (N)=N	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$=-3.507 bp_1 (V)=START	$\heartsuit_2(V)$= bp_2 (V) =	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$=-4.605 bp_1 (O)=START	$\heartsuit_2(O)$= bp_2 (O) =	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1})\right) + \heartsuit_{i-1}(y_{i-1})$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(??? * ???) + ???,
log(??? * ???) + ???,
log(??? * ???) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N → $\heartsuit_2$(V) = max(
log(e("flies" | V) * ???) + ???,

y_{i-1}=V → log(??? * ???) + ???,

y_{i-1}=O → log(??? * ???) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(e(“flies” | V) * q(V | N)) + ???,
log(??? * ???) + ???,
log(??? * ???) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),
log(??? * ???) + ???,
log(??? * ???) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(

log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),

log(e("flies" | V) * ???) + ???,

log(??? * ???) + ???

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(V) = \max($

$y_{i-1}=N$ → $\log(e(\text{“flies”} \mid V) * q(V \mid N)) + \heartsuit_1(N),$

$y_{i-1}=V$ → $\log(e(\text{“flies”} \mid V) * q(V \mid V)) + ???,$

$y_{i-1}=O$ → $\log(??? * ???) + ???$

$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=START	$\heartsuit_2(N)$=-3.835 bp_2 (N)=N	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$=-3.507 bp_1 (V)=START	$\heartsuit_2(V)$= bp_2 (V) =	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$=-4.605 bp_1 (O)=START	$\heartsuit_2(O)$= bp_2 (O) =	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(

log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),

log(e("flies" | V) * q(V | V)) + $\heartsuit_1$(V),

log(??? * ???) + ???

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),
log(e("flies" | V) * q(V | V)) + $\heartsuit_1$(V),
log(e("flies" | V) * ???) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\boxed{\heartsuit_2\text{(V)=} \;\; bp_2\text{ (V) =}}$	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(e(x_i \mid y_i) \times q(y_i \mid y_{i-1})\right) + \heartsuit_{i-1}(y_{i-1})$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),
log(e("flies" | V) * q(V | V)) + $\heartsuit_1$(V),
log(e("flies" | V) * q(V | O)) + ???
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(

log(e("flies" | V) * q(V | N)) + $\heartsuit_1$(N),

log(e("flies" | V) * q(V | V)) + $\heartsuit_1$(V),

log(e("flies" | V) * q(V | O)) + $\heartsuit_1$(O)

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(V) = max(
log(0.5 * 0.3) + -1.715,
log(0.5 * 0.1) + -3.507,
log(0.5 * 0.1) + -4.605
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N → $\heartsuit_2$(V) = max(

-3.612,

y_{i-1}=V → -6.503,

y_{i-1}=O → -7.601

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)= bp_2 (V) =	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(V) = -3.612$
$bp_2(V) = N$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=START	$\heartsuit_2(N)$=-3.835 bp_2 (N)=N	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$=-3.507 bp_1 (V)=START	$\heartsuit_2(V)$=-3.612 bp_2 (V)=N	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$=-4.605 bp_1 (O)=START	$\heartsuit_2(O)$= bp_2 (O) =	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(O) = max(

log(e("flies" | O) * q(O | N)) + $\heartsuit_1$(N),

log(e("flies" | O) * q(O | V)) + $\heartsuit_1$(V),

log(e("flies" | O) * q(O | O)) + $\heartsuit_1$(O)

)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

y_{i-1}=N

y_{i-1}=V

y_{i-1}=O

$\heartsuit_2$(O) = max(
log(0.0 * q(O | N)) + $\heartsuit_1$(N),
log(0.0 * q(O | V)) + $\heartsuit_1$(V),
log(0.0 * q(O | O)) + $\heartsuit_1$(O)
)

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N) = bp_3 (N) =	$\heartsuit_4$(N)= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)= bp_3 (V) =	$\heartsuit_4$(V)= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)= bp_2 (O) =	$\heartsuit_3$(O)= bp_3 (O) =	$\heartsuit_4$(O)= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1}\in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(O) = \max($

$y_{i-1}=N \rightarrow$ -inf,

$y_{i-1}=V \rightarrow$ -inf,

$y_{i-1}=O \rightarrow$ -inf

$)$

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=START	$\heartsuit_2(N)$=-3.835 bp_2 (N)=N	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$=-3.507 bp_1 (V)=START	$\heartsuit_2(V)$=-3.612 bp_2 (V)=N	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$=-4.605 bp_1 (O)=START	$\heartsuit_2(O)$= bp_2 (O) =	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

$\heartsuit_2(O)$ = -inf
$bp_2(O)$ = \

	fruit	flies	like	bananas	
	$\heartsuit_1(N)$=-1.715 bp_1 (N)=START	$\heartsuit_2(N)$=-3.835 bp_2 (N)=N	$\heartsuit_3(N)$ = bp_3 (N) =	$\heartsuit_4(N)$= bp_4 (N) =	
$\heartsuit_0$(START)=0	$\heartsuit_1(V)$=-3.507 bp_1 (V)=START	$\heartsuit_2(V)$=-3.612 bp_2 (V)=N	$\heartsuit_3(V)$= bp_3 (V) =	$\heartsuit_4(V)$= bp_4 (V) =	$\heartsuit_5$(STOP)= bp_5 (STOP) =
	$\heartsuit_1(O)$=-4.605 bp_1 (O)=START	$\boxed{\heartsuit_2(O)\text{=-inf}}$ bp_2 (O)=\	$\heartsuit_3(O)$= bp_3 (O) =	$\heartsuit_4(O)$= bp_4 (O) =	

10 minutes later

Quiz 6 - Problem 2

$$\heartsuit_i(y) = \max_{y_{i-1} \in L} \log\left(\underline{e(x_i \mid y_i)} \times \underline{q(y_i \mid y_{i-1})}\right) + \underline{\heartsuit_{i-1}(y_{i-1})}$$

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N)=-7.852 bp_3 (N)=O	$\heartsuit_4$(N)=-7.852 bp_4 (N)=V	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)=-5.955 bp_3 (V) =N	$\heartsuit_4$(V)=-inf bp_4 (V)=\	$\heartsuit_5$(STOP)=-9.461 bp_5 (STOP) =N
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=-inf bp_2 (O)=\	$\heartsuit_3$(O)=-6.425 bp_3 (O) =V	$\heartsuit_4$(O)=-8.075 bp_4 (O)=V	

Quiz 6 - Problem 2

fruit	flies	like	bananas	
$\heartsuit_1$(N)=log(0.3*0.6)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=max(log(0.3*0.4)-1.715, log(0.3*0.5)-3.507, log(0.3*0.7)-4.605)= -3.835 bp_2 (N) = N	$\heartsuit_3$(N)=max(log(0.1*0.4)-3.835, log(0.1*0.5)-3.612, log(0.1*0.7)-inf) = -6.608 bp_3 (N) = O	$\heartsuit_4$(N)=max(log(0.3*0.4)-6.608, log(0.3*0.5)-5.955, log(0.3*0.7)-6.425) = -7.852 bp_4 (N) = V	
$\heartsuit_1$(V)=log(0.1*0.3)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=max(log(0.5*0.3)-1.715, log(0.5*0.1)-3.507, log(0.5*0.1)-4.605)= -3.612 bp_2 (V) = N	$\heartsuit_3$(V)=max(log(0.4*0.3)-3.835, log(0.4*0.1)-3.612, log(0.4*0.1)-inf)= -5.955 bp_3 (V) = N	$\heartsuit_4$(V)=max(log(0.0)-6.608, log(0.0)-5.955, log(0.0)-6.425) = -inf bp_4 (V) = \	$\heartsuit_5$(STOP)=max(log(0.2)-7.852, log(0.2)-inf, log(0.1)-8.075) = -9.461 bp_5 (STOP) = N
$\heartsuit_1$(O)=log(0.1*0.1)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=max(log(0.0)-1.715, log(0.0)-3.507, log(0.0)-4.605) = -inf bp2 (O) = \	$\heartsuit_3$(O)=max(log(0.3*0.1)-3.835, log(0.3*0.2)-3.612, log(0.3*0.1)-inf) = -6.425 bp_3 (O) = V	$\heartsuit_4$(O)=max(log(0.6*0.1)-6.608, log(0.6*0.2)-5.955, log(0.6*0.1)-6.425) = -8.075 bp_4 (O) = V	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

	fruit	flies	like	bananas	
	♡$_1$(N)=-1.715 bp$_1$ (N)=START	♡$_2$(N)=-3.835 bp$_2$ (N)=N	♡$_3$(N)=-7.852 bp$_3$ (N)=O	♡$_4$(N)=-7.852 bp$_4$ (N)=V	
♡$_0$(START)=0	♡$_1$(V)=-3.507 bp$_1$ (V)=START	♡$_2$(V)=-3.612 bp$_2$ (V)=N	♡$_3$(V)=-5.955 bp$_3$ (V) =N	♡$_4$(V)=-inf bp$_4$ (V)=\	♡$_5$(STOP)=-9.461 bp$_5$ (STOP) =N
	♡$_1$(O)=-4.605 bp$_1$ (O)=START	♡$_2$(O)=-inf bp$_2$ (O)=\	♡$_3$(O)=-6.425 bp$_3$ (O) =V	♡$_4$(O)=-8.075 bp$_4$ (O)=V	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

Follow the backpointer!

	fruit	flies	like	bananas	
	$\heartsuit_1(N)=-1.715$ $bp_1(N)=START$	$\heartsuit_2(N)=-3.835$ $bp_2(N)=N$	$\heartsuit_3(N)=-7.852$ $bp_3(N)=O$	$\heartsuit_4(N)=-7.852$ $bp_4(N)=V$	
$\heartsuit_0(START)=0$	$\heartsuit_1(V)=-3.507$ $bp_1(V)=START$	$\heartsuit_2(V)=-3.612$ $bp_2(V)=N$	$\heartsuit_3(V)=-5.955$ $bp_3(V)=N$	$\heartsuit_4(V)=-inf$ $bp_4(V)=\backslash$	$\heartsuit_5(STOP)=-9.461$ $bp_5(STOP)=N$
	$\heartsuit_1(O)=-4.605$ $bp_1(O)=START$	$\heartsuit_2(O)=-inf$ $bp_2(O)=\backslash$	$\heartsuit_3(O)=-6.425$ $bp_3(O)=V$	$\heartsuit_4(O)=-8.075$ $bp_4(O)=V$	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N)=-7.852 bp_3 (N)=O	$\heartsuit_4$(N)=-7.852 bp_4 (N)=V	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)=-5.955 bp_3 (V) =N	$\heartsuit_4$(V)=-inf bp_4 (V)=\	$\heartsuit_5$(STOP)=-9.461 bp_5 (STOP) =N
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=-inf bp_2 (O)=\	$\heartsuit_3$(O)=-6.425 bp_3 (O) =V	$\heartsuit_4$(O)=-8.075 bp_4 (O)=V	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N)=-7.852 bp_3 (N)=O	$\heartsuit_4$(N)=-7.852 bp_4 (N)=V	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)=-5.955 bp_3 (V) =N	$\heartsuit_4$(V)=-inf bp_4 (V)=\	$\heartsuit_5$(STOP)=-9.461 bp_5 (STOP) =N
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=-inf bp_2 (O)=\	$\heartsuit_3$(O)=-6.425 bp_3 (O) =V	$\heartsuit_4$(O)=-8.075 bp_4 (O)=V	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N)=-7.852 bp_3 (N)=O	$\heartsuit_4$(N)=-7.852 bp_4 (N)=V	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)=-5.955 bp_3 (V) =N	$\heartsuit_4$(V)=-inf bp_4 (V)=\	$\heartsuit_5$(STOP)=-9.461 bp_5 (STOP) =N
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=-inf bp_2 (O)=\	$\heartsuit_3$(O)=-6.425 bp_3 (O) =V	$\heartsuit_4$(O)=-8.075 bp_4 (O)=V	

Quiz 6 - Problem 3

What is the decoded label sequence? Divide labels by space in your answer, e.g. (V V N O).

N N V N

	fruit	flies	like	bananas	
	$\heartsuit_1$(N)=-1.715 bp_1 (N)=START	$\heartsuit_2$(N)=-3.835 bp_2 (N)=N	$\heartsuit_3$(N)=-7.852 bp_3 (N)=O	$\heartsuit_4$(N)=-7.852 bp_4 (N)=V	
$\heartsuit_0$(START)=0	$\heartsuit_1$(V)=-3.507 bp_1 (V)=START	$\heartsuit_2$(V)=-3.612 bp_2 (V)=N	$\heartsuit_3$(V)=-5.955 bp_3 (V) =N	$\heartsuit_4$(V)=-inf bp_4 (V)=\	$\heartsuit_5$(STOP)=-9.461 bp_5 (STOP) =N
	$\heartsuit_1$(O)=-4.605 bp_1 (O)=START	$\heartsuit_2$(O)=-inf bp_2 (O)=\	$\heartsuit_3$(O)=-6.425 bp_3 (O) =V	$\heartsuit_4$(O)=-8.075 bp_4 (O)=V	

Q & A