# Linguistic Structure Prediction

CSE 447 / 517 Feb 24th, 2022 (Week 8)

### Logistics

- A7 is due tomorrow (Friday, 2/25).

### Agenda

- Syntax
  - What is syntax?
  - Syntactic Constituents
  - CKY
  - Quiz 7, part I: CKY walkthrough
- Semantics
  - What is semantics?
  - Compositional semantics and λ-calculus
- Q&A



### What Is Syntax?

Linguistic definition: the study of how phrases and sentences are structured

(If this piques your interest, the Linguistics department has several classes that discuss syntax at length; see also Emily Bender's book on <u>linguistic fundamentals</u> <u>for NLP</u>.)



### Syntactic Constituents

Running example: *My grumpy cat eats organic tuna with gusto.* 

**Constituents** are groups of words (e.g., *my grumpy cat*, *organic tuna*) that:

- Can move (It is organic tuna that my grumpy cat eats.)
- Can be coordinated (My grumpy cat eats organic tuna and Pacific salmon.)
- Can answer a question (*What does my grumpy cat eat? Organic tuna.*)

### Syntactic Constituents

Running example: My grumpy cat eats organic tuna with gusto.

#### **Types of constituents**

-

. . .

- Noun Phrase (NP) my grumpy cat, organic tuna
- Verb Phrase (VP) *eats organic tuna with gusto*
- Prepositional Phrase (PP) with gusto

### **Constituents and Recursion**

this is the house

. . .

this is the house that Jack built

this is the cat that lives in the house that Jack built

this is the dog that chased the cat that lives in the house that Jack built

### Context-Free Grammars (CFG)

A context-free grammar consists of:

- Finite set of nonterminals  ${\cal N}$
- Start symbol S
- Finite set of terminals (words)  $\Sigma$
- Production rule set  $\mathcal{R}$ , containing rules N  $\rightarrow \alpha$ 
  - N nonterminal from  ${\cal N}$
  - $\alpha$  sequence of 0 or more terminals/nonterminals
    - Chomsky normal form  $\alpha$  must be either a single terminal or two nonterminals

### **Phrase Structure Trees**

We can use the rules from our CFG to build a phrase structure tree for a given sentence. This represents both the **syntactic structure** of the sentence and its **derivation** from our CFG.



### Syntax in NLP

Given a CFG and a sentence **x**:

**Recognition** - is *x* in the CFG?

**Parsing** - how can we generate *x* from the rules of the CFG?

### PCFGs and CKY

Slides from Yejin Choi, Chris Manning, Dan Klein, Michael Collins, Luke Zettlemoyer, Ray Mooney, and Graham Neubig

### PCFG: Probabilistic Context Free Grammar

- A context-free grammar is a tuple <*N*, Σ, *S*, *R*>
  - **N** : the set of non-terminals
    - Phrasal categories: S, NP, VP, ADJP, etc.
    - Parts-of-speech (pre-terminals): NN, JJ, DT, VB, etc.
  - Σ : the set of terminals (the words)
  - S : the start symbol
    - Often written as ROOT or TOP
    - Not usually the sentence non-terminal S
  - *R* : the set of rules
    - Of the form  $X \rightarrow Y_1 Y_2 \dots Y_n$ , with  $X \in N$ ,  $n \ge 0$ ,  $Y_i \in (N \cup \Sigma)$
    - Examples:  $S \rightarrow NP VP$ ,  $VP \rightarrow VP CC VP$
- A PCFG adds a distribution q:
  - Probability q(r) for each  $r \in R$ , such that for all  $X \in N$ :

$$\sum_{\alpha \to \beta \in R: \alpha = X} q(\alpha \to \beta) = 1$$

### (Part of) A PCFG

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	0.5 0.3
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$	0.5 0.3
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$ $0.2  0.3  0.3$	0.5 0.3

### (Part of) A PCFG



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book \mid include \mid prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0 ( 0 1 0 05	
0.6 0.1 0.05	
0.6  0.1  0.05 NP $\rightarrow$ Det Nominal	0.6
0.6  0.1  0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal	0.6   <b>money</b>
0.6  0.1  0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal 0.03  0.15  0.06	0.6   money 0.06
0.6  0.1  0.05 $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ 0.03  0.15  0.06 $Nominal \rightarrow Nominal Nominal$	0.6   money 0.06 0.2
0.6  0.1  0.05 $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ 0.03  0.15  0.06 $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$	0.6   <b>money</b> 0.06 0.2 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$	0.6   <b>money</b> 0.06 0.2 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$	0.6   money 0.06 0.2 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$ $VP \rightarrow Verb NP$	0.6   money 0.06 0.2 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	0.6   <b>money</b> 0.06 0.2 0.5 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$	0.6   <b>money</b> 0.06 0.2 0.5 0.5 0.3
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$ $0.2 \ 0.3 \ 0.3$	0.6   money 0.06 0.2 0.5 0.5 0.5

~ ~



 $\mathbf{PP} \rightarrow \mathbf{Prep NP} \qquad 1.0$ 

### Why do we need CKY?

- Find the best tree t\* such that t\* = argmax P(t) over all possible trees.
- It's a decoding algorithm (analogy: Viterbi)
- Polynomial time (Dynamic programming)

# Dynamic Programming

 We will store: score of the max parse of x<sub>i</sub> to x<sub>j</sub> with root non-terminal X

$$\pi(i,j,X)$$

• So we can compute the most likely parse:

$$\pi(1, n, S) = \max_{t \in \mathcal{T}_G(s)} p(t)$$

• Via the recursion:

$$\pi(i,j,X) = \max_{\substack{X \to YZ \in R, \\ s \in \{i\dots(j-1)\}}} \left( q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z) \right)$$

• With base case:

$$\pi(i, i, X) = \begin{cases} q(X \to x_i) & \text{if } X \to x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

### **CKY Algorithm**

- Input: a sentence  $s = x_1 .. x_n$  and a PCFG =  $\langle N, \Sigma, S, R, q \rangle$
- Initialization: For i = 1 ... n and all X in N

$$\begin{array}{lll} \mbox{Bottom-up:}\\ \mbox{Starting from (i, i, X)} & \pi(i, i, X) & = \left\{ \begin{array}{ll} q(X \rightarrow x_i) & \mbox{if } X \rightarrow x_i \in R \\ 0 & \mbox{otherwise} \end{array} \right. \\ \mbox{otherwise} \end{array} \\ \mbox{For } i = 1 \hdots (n-1) & [\mbox{iterate all phrase lengths}] \\ \mbox{error } i = 1 \hdots (n-1) & [\mbox{iterate all phrases of length } i + 1] \end{array} \\ \mbox{By this point, every span} & \mbox{For all X in N} & [\mbox{iterate all non-terminals}] \\ \mbox{with length < I is already computed} & \pi(i, j, X) = \max_{\substack{X \rightarrow YZ \in R, \\ s \in \{i \dots (j-1)\} \end{array}} (q(X \rightarrow YZ) \times \pi(i, s, Y) \times \pi(s + 1, j, Z)) \\ \mbox{Previous states:} \\ \mbox{Best score for span (i, s) and non-terminal Y, and best score for span(s+1, j) and non-terminal Z} \\ \mbox{bp}(i, j, X) = \arg\max_{\substack{X \rightarrow YZ \in R, \\ s \in \{i \dots (j-1)\} \end{array}} (q(X \rightarrow YZ) \times \pi(i, s, Y) \times \pi(s + 1, j, Z)) \\ \mbox{For i = 1 } \left\{ \begin{array}{l} m_{i}(X) & m_{i}(X) = m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) = m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) & m_{i}(X) \\ \mbox{for i = 1 } m_{i}(X) \\ \mbox{for$$

### Quiz 7, Part I CKY

1. For the tree you build, what is the nonterminal for the constituent "prefer the flight to Houston"?

2. If "prefer" has a "Verb" tag, what could be a possible nonterminal for the constituent "the flight to Houston"?

3. If "prefer the flight" has a "VP" tag, what could be a possible nonterminal for the constituent "to Houston"?

### CKY Walkthrough: the Chart



Book the flight through Houston Start: Book the flight through Houston

### **CKY Walkthrough**





$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
02 03 03	
0.2 0.3 0.3	1.0

### Compute the (Max) Scores (2)



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	0.5
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
$Prep \rightarrow through \mid to \mid from$	
0.2 0.3 0.3	
	1.0



Structure	Sequence	Tree		
Algorithm	Viterbi	CKY (or CYK)		
State space	Tokens  x  Tags	Spans  x  Symbols  = O(N^2* Symbols )		
Time complexity	Tokens  x  Tags ^2	Spans  x  Tokens  x  Rules  = O(N^3* Rules )		
Filling the chart	Left-to-right	Bottom-up		
Recursive definition	$\pi(i, y_i) = \max_{y_{i-1}} e(x_i   y_i) q(y_i   y_{i-1})$	$)\pi(i-1,y_{i-1})$		
	$\pi(i,j,X) = \max_{\substack{X \to YZ \in R, \\ s \in \{i\dots(j-1)\}}} (q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z))$			



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
NID D ( NI I I	06
$NP \rightarrow Det Nominal$	0.0
$NP \rightarrow Det Nominal$ Nominal $\rightarrow book   flight   meal$	money
$NP \rightarrow Det Nominal Nominal \rightarrow book   flight   meal 0.03 0.15 0.06$	money 0.06
$ \begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \end{array} $	0.0   money 0.06 0.2
$ \begin{array}{c} NP \rightarrow Det Nominal \\ Nominal \rightarrow book \mid flight \mid meal \\ 0.03  0.15  0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \end{array} $	money 0.06 0.2 0.5
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$	money 0.06 0.2 0.5
$\begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \end{array}$	money 0.06 0.2 0.5
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$	0.0 <b>money</b> 0.06 0.2 0.5 0.5
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	0.0 0.06 0.2 0.5 0.5 0.3
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$	<b>money</b> <b>0.06</b> 0.2 0.5 0.5 0.3
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$ $0.2  0.3  0.3$	<pre>0.0   money 0.06 0.2 0.5 0.5 0.5 0.3</pre>

Ene	d:	_		_		
	<u> </u>	prefer	the	flight	to	Houston
Start:	NP: .1					
	prefer	S: .006 Verb: .06				
		the	Det: .6			
			flight	Nominal: .15		
				to	Prep: .3	
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book \mid include \mid prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VPPP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston   NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book flight meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb→ book   include   prefer	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$VP \rightarrow VP PP$	0.3
<b>Prep</b> $\rightarrow$ through to from	
0.2 0.3 0.3	

En	d:					
	1	prefer	the	flight	to	Houston
Start:	NP: .1					
	prefer	S: .006 Verb: .06				
		the	Det: .6			
			flight	Nominal: .15		
				to	Prep: .3	
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
	0.00
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP	0.2 0.5
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer	0.2 0.5
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06	0.2 0.5
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP	0.2 0.5 0.5
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP	0.2 0.5 0.5 0.3
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP Prep $\rightarrow$ through   to   from	0.2 0.5 0.5 0.3
$\begin{array}{l} \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \text{VP} \rightarrow \text{Verb NP} \\ \text{VP} \rightarrow \text{VPPP} \\ \text{Prep} \rightarrow \text{through} \mid \text{to} \mid \text{from} \\ 0.2  0.3  0.3 \end{array}$	0.2 0.5 0.5 0.3

En	d:					
	<u> </u>	prefer	the	flight	to	Houstor
Start:	NP: .1					
	prefer	S: .006 Verb: .06				
		the	Det: .6			
			flight	Nominal: .15		
				to	Prep: .3	
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	100
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
$PP \rightarrow Prep NP$	1.0



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	and an and
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb $\rightarrow$ book   include   prefer	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ through   to   from	
0.2 0.2 0.2	
0.2 0.3 0.3	

En	d:					
	1	prefer	the	flight	to	Houstor
Start:	NP: .1					
	prefer	S: .006 Verb: .06				
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		
			flight	Nominal: .15		
				to	Prep: .3	
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
	<b>^</b> /
$NP \rightarrow Det Nominal$	0.6
$NP \rightarrow Det Nominal$ Nominal $\rightarrow book   flight   meal$	0.6
$NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$	0.6   money 0.06
$\begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \end{array}$	0.8   money 0.06 0.2
$\begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \end{array}$	0.8   money 0.06 0.2 0.5
$\begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \end{array}$	0.6   money 0.06 0.2 0.5
$\begin{array}{c} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \end{array}$	0.6   money 0.06 0.2 0.5
$\begin{array}{l} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \text{VP} \rightarrow \text{Verb} \text{NP} \end{array}$	0.6   money 0.06 0.2 0.5
$\begin{array}{l} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \text{VP} \rightarrow \text{Verb} \text{NP} \\ \text{VP} \rightarrow \text{VPPP} \end{array}$	0.6   money 0.06 0.2 0.5 0.5 0.5
$\begin{array}{l} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \text{VP} \rightarrow \text{Verb} \text{NP} \\ \text{VP} \rightarrow \text{VPPP} \\ \text{Prep} \rightarrow \text{through} \mid \text{to} \mid \text{from} \end{array}$	0.6   money 0.06 0.2 0.5 0.5 0.5
$\begin{array}{l} \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \text{Nominal} \rightarrow \text{Nominal PP} \\ \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \text{VP} \rightarrow \text{Verb NP} \\ \text{VP} \rightarrow \text{VP PP} \\ \text{Prep} \rightarrow \text{through} \mid \text{to} \mid \text{from} \\ 0.2  0.3  0.3 \end{array}$	0.6   money 0.06 0.2 0.5 0.5 0.5 0.3



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
$PP \rightarrow Prep NP$	1.0

Houstor
PP: 1.0 *.3*.16 = <b>.048</b>
NP: .16
P *.

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	1.201.01
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	
riommar / riommar r	0.5
Verb $\rightarrow$ book   include   prefer	0.5
$Verb \rightarrow book \mid include \mid prefer \\ 0.5  0.04  0.06$	0.5
Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP	0.5
Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP	0.5 0.5 0.3
Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP Prep $\rightarrow$ through   to   from	0.5 0.5 0.3
Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP Prep $\rightarrow$ through   to   from 0.2 0.3 0.3	0.5 0.5 0.3



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	<b>•</b> (
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
$Verb \rightarrow book \mid include \mid prefer$	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$VP \rightarrow VP PP$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
$PP \rightarrow Prep NP$	1.0

_			
	0		
		U	_
		-	_

	l	prefer	the	flight	to	Houston
Start:	NP: .1					
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 <b>=.000162</b> VP: . <b>00162</b>		
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		
			flight	Nominal: .15		
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb→ book   include   prefer	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
$PP \rightarrow Prep NP$	1.0

_		
	10	
		-

	<u> </u>	prefer	the	flight	to	Houston
Start:	NP: .1					
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 <b>=.000162</b> VP: . <b>00162</b>		
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		
			flight	Nominal: .15		Nominal: .5*.15*.04 8= <b>.0036</b>
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book \mid include \mid prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
Nominal $\rightarrow$ book   flight   meal 0.03 0.15 0.06	money 0.06
Nominal → book   flight   meal 0.03 0.15 0.06 Nominal → Nominal Nominal	money 0.06 0.2
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal}  \hline \text{Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal}  PP \end{array}$	money 0.06 0.2 0.5
$\begin{array}{l} Nominal \rightarrow book \mid flight \mid meal \\ 0.03  0.15  0.06 \\ \hline Nominal \rightarrow Nominal \ Nominal \\ \hline Nominal \rightarrow Nominal \ PP \\ \hline Verb \rightarrow book \mid include \mid prefer \end{array}$	<b>money</b> 0.06 0.2 0.5
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal}  \hline \text{Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal}  PP \\ \hline \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \hline \end{array}$	money 0.06 0.2 0.5
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal}  \hline \text{Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal}  PP \\ \hline \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \hline \text{VP} \rightarrow \text{Verb}  NP \\ \end{array}$	money 0.06 0.2 0.5
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal}  \hline \text{Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal}  PP \\ \hline \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \hline \text{VP} \rightarrow \text{Verb}  \text{NP} \\ \hline \text{VP} \rightarrow \text{VP}  PP \end{array}$	money 0.06 0.2 0.5 0.5 0.3
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal PP} \\ \hline \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \hline \text{VP} \rightarrow \text{Verb} \text{NP} \\ \hline \text{VP} \rightarrow \text{VPPP} \\ \hline \text{Prep} \rightarrow \text{through} \mid \text{to} \mid \text{from} \end{array}$	<b>money</b> 0.06 0.2 0.5 0.5 0.3
$\begin{array}{l} \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03  0.15  0.06 \\ \hline \text{Nominal} \rightarrow \text{Nominal}  \hline \text{Nominal} \\ \hline \text{Nominal} \rightarrow \text{Nominal}  PP \\ \hline \text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{prefer} \\ 0.5  0.04  0.06 \\ \hline \text{VP} \rightarrow \text{Verb}  \text{NP} \\ \hline \text{VP} \rightarrow \text{VPP} \\ \hline \text{Prep} \rightarrow \text{through} \mid \text{to} \mid \text{from} \\ 0.2  0.3  0.3 \\ \hline \end{array}$	money 0.06 0.2 0.5 0.5 0.3



$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer 0.01 0.004 0.006$	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I$   he   she   me	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb→ book   include   prefer	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
	4.0

#### End:

	1	prefer	the	flight	to	Houston
Start:	NP: .1			S: .8*.1*.001 62= .00013		
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 <b>=.000162</b> VP: . <b>00162</b>		
		the	Det: .6	NP: .6*.6 *.15 <b>= .054</b>		
			flight	Nominal: .15		Nominal: .5*.15*.04 8= <b>.0036</b>
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb→ book   include   prefer	
0.5 0.04 0.06	
$VP \rightarrow Verb NP$	0.5
$\mathbf{VP} \rightarrow \mathbf{VP} \mathbf{PP}$	0.3
<b>Prep</b> $\rightarrow$ <b>through</b>   <b>to</b>   <b>from</b>	
0.2 0.3 0.3	
$PP \rightarrow Prep NP$	1.0

#### End:

	<u> </u>	prefer	the	flight	to	Houston
Start:	rt: I NP:.1			S: .8*.1*.001 62= .00013		
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 <b>=.000162</b> VP: . <b>00162</b>		
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		NP: .6*.6*.003 6 = .001296
			flight	Nominal: .15		Nominal: .5*.15*.04 8 <b>=.0036</b>
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				Н	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal hool flight mool	money
Nominal $\rightarrow$ Dook   inglit   mean	Imonoj
$\begin{array}{r} \text{Nominal} \rightarrow \text{book} \mid \text{light} \mid \text{light} \mid \text{light} \\ 0.03  0.15  0.06 \end{array}$	0.06
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal	<b>0.06</b> 0.2
Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP	<b>0.06</b> 0.2 0.5
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer	<b>0.06</b> 0.2 0.5
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06	<b>0.06</b> 0.2 0.5
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP	<b>0.06</b> 0.2 0.5
Nominal $\rightarrow$ book   night   mean 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP	<b>0.06</b> 0.2 0.5 0.5 0.3
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP Prep $\rightarrow$ through   to   from	<b>0.06</b> 0.2 0.5 0.5 0.3
Nominal $\rightarrow$ book   hight   heat 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP VP $\rightarrow$ VP PP Prep $\rightarrow$ through   to   from 0.2 0.3 0.3	<b>0.06</b> 0.2 0.5 0.5 0.3

#### End:

	1	prefer	the	flight	to	Houston
Start:				S: .8*.1*.001 62= .00013		
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 <b>=.000162</b> VP: . <b>00162</b>		
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		NP: .6*.6*.003 6 = .001296
			flight	Nominal: .15		Nominal: .5*.15*.04 8 <b>=.0036</b>
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				F	louston	NP: .16

$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
$NP \rightarrow Det Nominal$	0.6
Nominal $\rightarrow$ book   flight   meal	money
0.03 0.15 0.06	0.06
Nominal $\rightarrow$ Nominal Nominal	0.2
Nominal $\rightarrow$ Nominal PP	0.5
Verb $\rightarrow$ book   include   prefer	
0 5 0 04 0 06	
0.0 0.04 0.00	
$VP \rightarrow Verb NP$	0.5
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	0.5 0.3
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$	0.5 0.3
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$ $0.2  0.3  0.3$	0.5 0.3





$S \rightarrow NP VP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$\mathbf{Det} \rightarrow \mathbf{the} \mid \mathbf{a} \mid \mathbf{an}$	
0 ( 01 005	
0.6 0.1 0.05	
0.6  0.1  0.05 NP $\rightarrow$ Det Nominal	0.6
0.6  0.1  0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal	0.6   <b>money</b>
$0.6  0.1  0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$	0.6   money 0.06
$ \begin{array}{c} 0.6 & 0.1 & 0.05 \\ \text{NP} \rightarrow \text{Det Nominal} \\ \text{Nominal} \rightarrow \text{book} \mid \text{flight} \mid \text{meal} \\ 0.03 & 0.15 & 0.06 \\ \text{Nominal} \rightarrow \text{Nominal Nominal} \end{array} $	0.6   <b>money</b> 0.06 0.2
0.6 0.1 0.05 $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ 0.03 0.15 0.06 $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$	0.6   <b>money</b> 0.06 0.2 0.5
0.6 0.1 0.05 $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ 0.03 0.15 0.06 $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$	0.6   <b>money</b> 0.06 0.2 0.5
$0.6 \ 0.1 \ 0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03 \ 0.15 \ 0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5 \ 0.04 \ 0.06$	0.6   <b>money</b> 0.06 0.2 0.5
0.6 0.1 0.05 $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ 0.03 0.15 0.06 $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ 0.5 0.04 0.06 $VP \rightarrow Verb NP$	0.6   <b>money</b> 0.06 0.2 0.5
$0.6  0.1  0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	0.6   money 0.06 0.2 0.5 0.5
$ \begin{array}{c} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book   flight   meal \\ 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \\ Verb \rightarrow book   include   prefer \\ 0.5 & 0.04 & 0.06 \\ \hline VP \rightarrow Verb NP \\ VP \rightarrow VP PP \\ \hline Prep \rightarrow through   to   from \\ \end{array} $	0.6   <b>money</b> 0.06 0.2 0.5 0.5
$0.6  0.1  0.05$ $NP \rightarrow Det Nominal$ $Nominal \rightarrow book   flight   meal$ $0.03  0.15  0.06$ $Nominal \rightarrow Nominal Nominal$ $Nominal \rightarrow Nominal PP$ $Verb \rightarrow book   include   prefer$ $0.5  0.04  0.06$ $VP \rightarrow Verb NP$ $VP \rightarrow VP PP$ $Prep \rightarrow through   to   from$ $0.2  0.3  0.3$	0.6   money 0.06 0.2 0.5 0.5

End:						
	I	prefer	the	flight	to	Houston
Start:				S: .8*.1*.001 62= .00013		S: .8*.1*.000 039 <b>=3e-6</b>
	prefer	S: .006 Verb: .06		S:.05*.06 *.054 = <b>.000162</b> VP: . <b>00162</b>		S:4e-5 VP:.5*.06 *.001296 = .000039
		the	Det: .6	NP: .6*.6 *.15 = <b>.054</b>		NP: .6*.6*.003 6 = .001296
			flight	Nominal: .15		Nominal: .5*.15*.04 8 <b>=.0036</b>
				to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
				F	louston	NP: .16

	0.0
$S \rightarrow NPVP$	0.8
$S \rightarrow X1 VP$	0.1
$X1 \rightarrow Aux NP$	1.0
$S \rightarrow book   include   prefer$	
0.01 0.004 0.006	
$S \rightarrow Verb NP$	0.05
$S \rightarrow VP PP$	0.03
$NP \rightarrow I \mid he \mid she \mid me$	
0.1 0.02 0.02 0.06	
$NP \rightarrow Houston \mid NWA$	
0.16 .04	
$Det \rightarrow the \mid a \mid an$	
0.6 0.1 0.05	
$0.6  0.1  0.05$ $NP \rightarrow Det Nominal$	0.6
0.6 0.1 0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal	0.6   <b>money</b>
0.6 0.1 0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal 0.03 0.15 0.06	0.6   money 0.06
0.6 0.1 0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal	0.6   <b>money</b> 0.06 0.2
$\begin{array}{ccc} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book   flight   meal \\ 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \end{array}$	0.6   <b>money</b> 0.06 0.2 0.5
0.6 0.1 0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer	0.6   <b>money</b> 0.06 0.2 0.5
$\begin{array}{cccc} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book &   flight   meal \\ & 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \\ Verb \rightarrow book &   include   prefer \\ & 0.5 & 0.04 & 0.06 \\ \end{array}$	0.6   <b>money</b> 0.06 0.2 0.5
0.6 0.1 0.05 NP $\rightarrow$ Det Nominal Nominal $\rightarrow$ book   flight   meal 0.03 0.15 0.06 Nominal $\rightarrow$ Nominal Nominal Nominal $\rightarrow$ Nominal PP Verb $\rightarrow$ book   include   prefer 0.5 0.04 0.06 VP $\rightarrow$ Verb NP	0.6   <b>money</b> 0.06 0.2 0.5
$\begin{array}{cccc} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book   flight   meal \\ & 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \\ Verb \rightarrow book   include   prefer \\ & 0.5 & 0.04 & 0.06 \\ VP \rightarrow Verb NP \\ VP \rightarrow VP PP \end{array}$	0.6   <b>money</b> 0.06 0.2 0.5 0.5
$\begin{array}{cccc} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book   flight   meal \\ 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \\ Verb \rightarrow book   include   prefer \\ 0.5 & 0.04 & 0.06 \\ VP \rightarrow Verb NP \\ VP \rightarrow VP PP \\ Prep \rightarrow through   to   from \end{array}$	0.6   <b>money</b> 0.06 0.2 0.5 0.5 0.3
$\begin{array}{cccc} 0.6 & 0.1 & 0.05 \\ NP \rightarrow Det Nominal \\ Nominal \rightarrow book   flight   meal \\ 0.03 & 0.15 & 0.06 \\ Nominal \rightarrow Nominal Nominal \\ Nominal \rightarrow Nominal PP \\ Verb \rightarrow book   include   prefer \\ 0.5 & 0.04 & 0.06 \\ VP \rightarrow Verb NP \\ VP \rightarrow VP PP \\ Prep \rightarrow through   to   from \\ 0.2 & 0.3 & 0.3 \\ \end{array}$	0.6   <b>money</b> 0.06 0.2 0.5 0.5 0.3



VP









### Quiz 7, Part I CKY

1. For the tree you build, what is the nonterminal for the constituent "prefer the flight to Houston"?

VP



### Quiz 7, Part I CKY

1. For the tree you build, what is the nonterminal for the constituent "prefer the flight to Houston"?

#### VP

2. If "prefer" has a "Verb" tag, what could be a possible nonterminal for the constituent "the flight to Houston"?

NP



		Enc	d:					
			I	prefer	the	flight	to	Houston
Qı	iiz 7, Part I CKY	t: 1	NP: .1			S: .8*.1*.00 162= .00013		
1.	For the tree you build, what is the the constituent "prefer the flight to	nont Hou	terminal ıston"?	for S: .006 Verb: .06		S:.05*.06 *.054 = <b>.000162</b> VP: .00162		
	VP					and the second se		NP <sup>.</sup>
2.	If "prefer" has a "Verb" tag, what could be a possible nonterminal for the constituent "the flight to			Det: .6	NP: .6*.6 *.15 = <b>.054</b>		.6*.6*.00 36 = .001296	
	Houston"?	?				Nominal		Nominal:
	NP				flight	.15		.5*.15*.0 48= <b>.0036</b>
3.	If "prefer the flight" has a "VP" tag, possible nonterminal for the consti	wha tuer	at could nt "to Hc	be a puston"?		to	Prep: .3	PP: 1.0 *.3*.16 = <b>.048</b>
	PP					F	louston	NP: .16

## **Semantics**

### What Is Semantics?

- Syntax concerns the structure of sentences, i.e., does a sentence conform to the rules of a language?
  - PL parallel: "Does a program compile?"
- Semantics concerns the meaning of sentences, usually using syntax as a scaffold
  - PL parallel: "What is the output of a program?"
- There are syntactically well-formed yet semantically infelicitous sentences...
  - Famous example due to Chomsky: "Colorless green ideas sleep furiously."
  - "The present King of France is bald."
- ... and arguably semantically felicitous yet syntactically ill-formed ones
  - "Dog loyal animal."
    - Cf. "Dogs are loyal animals."

### **First-Order logic**

- ▶ **Term:** a constant (*ss*) or a variable
- ► Formula: defined inductively ...
  - ▶ If R is an n-ary relation and t<sub>1</sub>,...,t<sub>n</sub> are terms, then R(t<sub>1</sub>,...,t<sub>n</sub>) is a formula.
  - ▶ If  $\phi$  is a formula, then its negation,  $\neg \phi$ , is a formula.
  - If \u03c6 and \u03c6 are formulas, then binary logical connectives can be used to create formulas:
    - $\blacktriangleright \phi \wedge \psi$
    - $\blacktriangleright \phi \lor \psi$
    - $\blacktriangleright \phi \Rightarrow \psi$
  - If \u03c6 is a formula and v is a variable, then quantifiers can be used to create formulas:
    - Universal quantifier:  $\forall v, \phi$
    - **•** Existential quantifier:  $\exists v, \phi$

### Bonus Question $\rightarrow$ English Translation

Q3:  $\forall x, Quiz(x) \Rightarrow (Hard(x) \lor \neg Does(Adrian, x))$ 

A3: Every quiz is hard or is not done by Adrian.

Q4:  $\exists y, \forall x, Quiz(y) \Rightarrow \neg Does(x, y)$ 

A4: There exists some quizzes that no one does

OR not everything in the world is a quiz!

```
Q4': \exists y, \forall x, Quiz(y) \land \neg Does(x, y)
```

### Bonus English $\rightarrow$ FOL Translation

- "Every farmer who owns a donkey beats it."
- This type of sentence, called "donkey sentences", is well-known in semantic theory
- Answer in <u>https://en.wikipedia.org/wiki/Donkey\_sentence</u> (but try it first -- it's fun!)

### **Compositional Semantics**

- How do we get to FOL (or some other meaning representation) from natural language?
- Theory: compositionality
  - Human can't remember the meaning of all sentences, as there are infinitely many of them (e.g., recursion)
  - So sentence meanings are composed from smaller parts, with rules stitching them together
    - Often on top of the syntax tree
  - The most basic pieces of meaning come from the lexicon
- Tool: λ-calculus

### λ-Calculus

- Anonymized functions; like the lambdas in Python/Java/etc.
- Syntax of  $\lambda$ -calculus (which is in a sense arbitrary):  $\lambda v$  .  $\phi$ 
  - In Python: lambda v:  $\phi$
- Usually, instead of a value as the output (e.g., λx . x + 1), we have a statement (e.g., λx . x runs)
  - You can think of the statement as a "truth-condition" that evaluates to either true or false given the world state
  - So the output is just a special type of object, if you will
- In compositional semantics, as you walk up the tree, the argument(s) of the  $\lambda$ s get filled in and (necessarily) ending up with a arity/valency of 0 at the root

- Input: "John likes Mary"
- Goal: The input is true iff John likes Mary
  - Not so interesting in this case, but we still want to see how we derive this compositionally

- Input: "John likes Mary"
- Goal: The input is true iff John likes Mary
  - Not so interesting in this case, but we still want to see how we derive this compositionally
- Step 1: parse



- Input: "John likes Mary"
- Goal: The input is true iff John likes Mary
  - Not so interesting in this case, but we still want to see how we derive this compositionally
- Step 2: apply the lexicon



- Input: "John likes Mary"
- Goal: The input is true iff John likes Mary
  - Not so interesting in this case, but we still want to see how we derive this compositionally
- Step 3: compose



- Input: "John likes Mary"
- Goal: The input is true iff John likes Mary
  - Not so interesting in this case, but we still want to see how we derive this compositionally
- Step 3: compose



# Q & A