Natural Language Processing (CSE 517 & 447): Conditional Random Fields

Noah Smith

University of Washington
nasmith@cs.washington.edu

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Readings: Eisenstein (2019) 7 and 8

Motivation

Many tasks in NLP can be cast as **sequence labeling**, where each token (usually, word) gets its own label. Compare:

- Text classification: $\langle x_1, x_2, \ldots, x_n \rangle \mapsto y \in \mathcal{L}$
- Sequence labeling: $\langle x_1 \mapsto y_1, x_2 \mapsto y_2, \ldots, x_n \mapsto y_n \rangle$, each $y_i \in \mathcal{L}$
- Translation: $\boldsymbol{x} \mapsto \boldsymbol{y} \in \mathcal{V}^*_{target}$

Problems Typically Cast as Sequence Labeling

- supersense tagging (Ciaramita and Johnson, 2003)
- part-of-speech tagging (Church, 1988)
- morphosyntactic tagging (Habash and Rambow, 2005)
- segmentation into words (Sproat et al., 1996) or multiword expressions (Schneider et al., 2014)
- code switching (Solorio and Liu, 2008)
- dialogue acts (Stolcke et al., 2000)
- spelling correction (Kernighan et al., 1990)
- word alignment (Vogel et al., 1996)
- named entity recognition (Bikel et al., 1999)
- compression (Conroy and O'Leary, 2001)

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Ciaramita and Johnson (2003) and Ciaramita and Altun (2006) used a lexicon called WordNet to define 41 semantic classes for words.

- WordNet (Fellbaum, 1998) is a fascinating resource in its own right! See `http://wordnetweb.princeton.edu/perl/webwn` to get an idea.

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This represents a coarsening of the annotations in the Semcor corpus (Miller et al., 1993).

Example: *box*'s Thirteen Synonym Sets, Eight Supersenses

1. box: a (usually rectangular) container; may have a lid. "he rummaged through a box of spare parts"
2. box/loge: private area in a theater or grandstand where a small group can watch the performance. "the royal box was empty"
3. box/boxful: the quantity contained in a box. "he gave her a box of chocolates"
4. corner/box: a predicament from which a skillful or graceful escape is impossible. "his lying got him into a tight corner"
5. box: a rectangular drawing. "the flowchart contained many boxes"
6. box/boxwood: evergreen shrubs or small trees
7. box: any one of several designated areas on a ball field where the batter or catcher or coaches are positioned. "the umpire warned the batter to stay in the batter's box"
8. box/box seat: the driver's seat on a coach. "an armed guard sat in the box with the driver"
9. box: separate partitioned area in a public place for a few people. "the sentry stayed in his box to avoid the cold"
10. box: a blow with the hand (usually on the ear). "I gave him a good box on the ear"
11. box/package: put into a box. "box the gift, please"
12. box: hit with the fist. "I'll box your ears!"
13. box: engage in a boxing match.

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1. box: a (usually rectangular) container; may have a lid. "he rummaged through a box of spare parts" ⇝ N.ARTIFACT
2. box/loge: private area in a theater or grandstand where a small group can watch the performance. "the royal box was empty" ⇝ N.ARTIFACT
3. box/boxful: the quantity contained in a box. "he gave her a box of chocolates" ⇝ N.QUANTITY
4. corner/box: a predicament from which a skillful or graceful escape is impossible. "his lying got him into a tight corner" ⇝ N.STATE
5. box: a rectangular drawing. "the flowchart contained many boxes" ⇝ N.SHAPE
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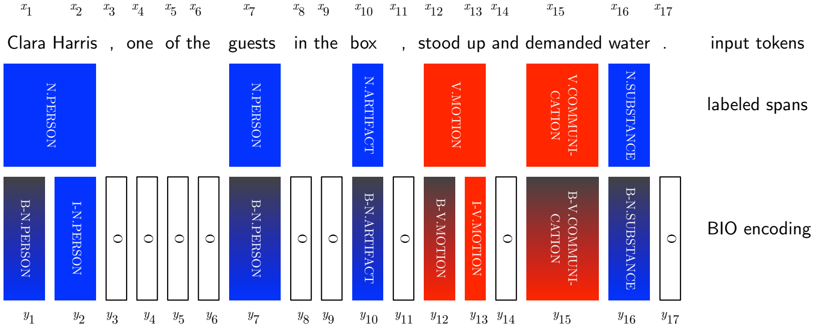
13. box: engage in a boxing match. ⇝ V.COMPETITION

Supersense Tagging Example



x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	
Clara	Harris	,	one	of	the	guests	in	the	box	,	stood	up	and	demanded	water	.	input tokens
N.PERSON						N.PERSON			N.ARTIFACT		V.MOTION			V.COMMUNICATION	N.SUBSTANCE		labeled spans
B-N.PERSON	I-N.PERSON	O	O	O	O	B-N.PERSON	O	O	B-N.ARTIFACT	O	B-V.MOTION	I-V.MOTION	O	B-V.COMMUNICATION	B-N.SUBSTANCE	O	BIO encoding
y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	

Observations

- Lots of subproblems: Which words have supersenses? Which words group together to form a multiword expression? For those that do, which supersense?
- Every word's label depends on the words around it, and their labels.
- Segmentation problems can be cast as sequence labeling (Ramshaw and Marcus, 1995):
  - Two labels, B and I, if every word must be in some segment
  - Three labels, B, I, and O, if some words are to be "discarded"
  - Variants for five labels (E for end, S for singleton), gaps/noncontiguous spans, and nesting, exist.

  Concatenate B, I, etc., with labels to get labeled segmentation.
- Some sequences of labels might be invalid under your theory/label semantics.
- Evaluation: usually precision, recall, and F_1 on labeled segments.

Big Abstraction: Linguistic Analysis

Every linguistic analyzer is comprised of:

1. Theoretical motivation from linguistics and/or the text domain
2. An algorithm that maps $\mathcal{V}^\dagger$ to some output space $\mathcal{Y}$.
   - Some $\mathcal{Y}$ are very specialized, but others, like the one we discuss here, show up again and again.
3. An implementation of the algorithm
   - Once upon a time: rule systems and crafted rules
   - More robust: supervised learning from annotated data
   - Today: unsupervised pretraining followed by supervised finetuning

Sequence Labeling

Problem statement: given a sequence of n words $\boldsymbol{x}$, assign each a label from $\mathcal{L}$. Let $L = |\mathcal{L}|$.

Every approach we see today will cast the problem as:

$$\hat{\boldsymbol{y}} = \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta})$$

Naïvely, that's a classification problem where the number of possible 'labels" (output sequences) depends on the input and is $O(L^n)$ in size!

Sequence Labeling v. 0: Local Classifiers

Define score of a word x_i getting label $y \in \mathcal{L}$ in context: $\mathrm{score}(\boldsymbol{x}, i, y; \boldsymbol{\theta})$, for example through a feature vector, $\mathbf{f}(\boldsymbol{x}, i, y)$. (Here, "$i$"' indicates the position of the input word to be classified.)

Train a classifier to decode locally, i.e.,

$$\begin{aligned}\hat{y}_i &= \operatorname*{argmax}_{y \in \mathcal{L}} \mathrm{score}(\boldsymbol{x}, i, y; \boldsymbol{\theta}) \\ &\overset{\text{MLR}}{=} \operatorname*{argmax}_{y \in \mathcal{L}} \boldsymbol{\theta}^\top \mathbf{f}(\boldsymbol{x}, i, y)\end{aligned}$$

The classifier is applied to each $x_1, x_2, \ldots$ in turn, but all the words can be made available at each position.

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We can do better when there are predictable relationships among labels.

Reflection

If we return to the original formulation,

$$\hat{\boldsymbol{y}} = \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta}),$$

how can we write "Score" in terms of the notation on the last slide?

Local Classifiers (v. 0)

Lightweight; no need to learn anything new! But labels can't affect each other.

Sequence Labeling v. 1: Sequential Classifiers

Define score of a word x_i getting label y in context, *including previous labels*: $\mathrm{score}(\boldsymbol{x}, i, \hat{\boldsymbol{y}}_{1:i-1}, y; \boldsymbol{\theta})$. (From here, we won't always write $\boldsymbol{\theta}$, but the dependence remains.)

Train a classifier, e.g.,

$$\hat{y}_i = \operatorname*{argmax}_{y \in \mathcal{L}} \mathrm{score}(\boldsymbol{x}, i, \hat{\boldsymbol{y}}_{1:i-1}, y)$$

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There is much literature on methods for training, and for decoding, with models like this. Important decoding method in NLP: beam search.

Beam Search for Sequential Classifiers

Input: $\boldsymbol{x}$ (length n), a sequential classifier's scoring function score, and beam width k

```
Let $H_0$ score hypotheses at position $0$, defining only $H_0(\langle\rangle) = 0$.
For $i \in \{1, \ldots, n\}$:
    ▶ Empty $C$.
    ▶ For each hypothesis $\hat{\boldsymbol{y}}_{1:i-1}$ scored by $H_{i-1}$:
        ▶ For each $y \in \mathcal{L}$, place new hypothesis
          $\hat{\boldsymbol{y}}_{1:i}y \rightarrow H_{i-1}(\hat{\boldsymbol{y}}_{1:i-1}) + \mathrm{score}(\boldsymbol{x}, i, \hat{\boldsymbol{y}}_{1:i-1}, y)$ into $C$.
    ▶ Let $H_i$ be the $k$-best scored elements of $C$.
```

Output: best scored element of H_n.

Notes on Beam Search for Sequential Classifiers

- Runtime is $O(n^2kL)$, space is $O(n^2k)$.
- You can improve runtime (e.g., to $O(nkL)$) if computation is shared across different i (often true with neural networks).
- Special cases:
  - $k = 1$ is greedy left-to-right decoding.
  - At $k = L^n$, you're doing brute force, exhaustive search.
- Generally: no guarantee.

Reflection

Suppose your label set is built out of BIO tags. For an output $\hat{\boldsymbol{y}}$ to be well-formed, it suffices to ensure that it contains no "OI" label bigrams.

How would you modify beam search to guarantee well-formedness?

Sequential Classifiers (v. 1)

Very powerful! Algorithms lack guarantees.

A Generative Approach

The next approach should remind you of language models. It assumes that labeled sequences are generated according to the following story:

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$$\begin{tikzcd}
x_1 & \\
y_1 \arrow[u] \arrow[r] & y_2
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\end{tikzcd}$$

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$$\begin{tikzcd}
x_1 & x_2 & x_3 & \\
y_1 \arrow[u] \arrow[r] & y_2 \arrow[u] \arrow[r] & y_3 \arrow[u] \arrow[r] & y_4
\end{tikzcd}$$

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$$\begin{tikzcd}
x_1 & & x_2 & & x_3 & & x_4 & & \\
y_1 \arrow[u] \arrow[r] & {} \arrow[r, phantom] & y_2 \arrow[u] \arrow[r] & {} \arrow[r, phantom] & y_3 \arrow[u] \arrow[r] & {} \arrow[r, phantom] & y_4 \arrow[u] \arrow[r] & {} \arrow[r, phantom] & \text{⯃}
\end{tikzcd}$$

$$y_5 \sim p_{transition}(Y \mid y_4)$$

Sequence Labeling v. 2: Hidden Markov Models

By convention, $y_{n+1} =$ ⯃ is always the “stop label.”

$$\begin{aligned}
p(\boldsymbol{X} = \boldsymbol{x}, \boldsymbol{Y} = \boldsymbol{y}) &= p_{start}(y_1) \cdot \\
&\qquad \prod_{i=1}^{n} p_{emission}(x_i \mid y_i) \cdot p_{transition}(y_{i+1} \mid y_i) \\
\hat{\boldsymbol{y}} &= \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} p(\boldsymbol{Y} = \boldsymbol{y} \mid \boldsymbol{X} = \boldsymbol{x}) \\
&= \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} p(\boldsymbol{X} = \boldsymbol{x}, \boldsymbol{Y} = \boldsymbol{y}) \\
&= \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} \log p(\boldsymbol{X} = \boldsymbol{x}, \boldsymbol{Y} = \boldsymbol{y})
\end{aligned}$$

We can solve the global decoding problem *exactly* (i.e., find the model-optimal $\hat{\boldsymbol{y}}$) in $O(nL^2)$ time and $O(nL)$ space using the Viterbi algorithm (more later).

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$p_{transition}$ is exactly a bigram (first-order Markov) model over labels.

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With classical HMMs, the parameters ($p_{transition}, p_{emission}, p_{start}$) have a closed form if you have labeled data! The hardest part is implementing the algorithm for choosing the "argmax" label sequence. Downside:

- You don't get to design or learn features.

Reflection

The runtime of the model-optimal decoding algorithm for HMMs depends quadratically on the size of $\mathcal{L}$. For some problems (e.g., supersense tagging) the label set can be large. Can you think of a way to trade the guarantee of model-optimality for speed, while still using the HMM?

Hidden Markov Models (v. 2)

Algorithmically beautiful; lack of features is unsatisfying.

Sequence Labeling v. 3

To endow HMMs with features, we can replace the "lookup" probabilities ($p_{transition}, p_{emission}, p_{start}$) with scoring functions.

This idea was explored by Berg-Kirkpatrick et al. (2010).

Classical HMM (v. 2):

$$\hat{\boldsymbol{y}} = \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} \log p_{start}(y_1) + \sum_{i=1}^{n} \left(\begin{array}{l} \log p_{emission}(x_i \mid y_i) \\ + \ \log p_{transition}(y_{i+1} \mid y_i) \end{array} \right)$$

This approach (v. 3):

$$\hat{\boldsymbol{y}} = \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} s_{start}(y_1) + \sum_{i=1}^{n} s_{emission}(x_i, y_i) + s_{transition}(y_i, y_{i+1})$$

Each "s" could be a linear scoring function (like in MLR), perhaps using word or label vectors. For now, I'm hiding the parameters of each s.

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- No part of the the scoring function looks at neighboring words.

V. 3

Brings features to HMMs, but learning is going to require more than just counting and normalizing.

Sequence Labeling v. 4

Let each scoring component ("s") "see" the whole input. By convention, $y_0 =$ ○ is always the "start label."

$$\hat{\boldsymbol{y}} = \operatorname*{argmax}_{\boldsymbol{y} \in \mathcal{L}^n} \overbrace{\sum_{i=0}^{n} s(\boldsymbol{x}, i, y_i, y_{i+1})}^{\text{Score}(\boldsymbol{x}, \boldsymbol{y})}$$

Note that $\boldsymbol{x}$ can have arbitrary length, so we need "s" functions that are capable of adapting to variable-length input.

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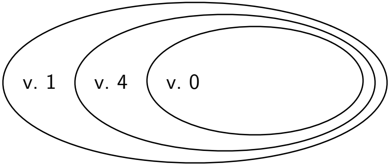
- Decoding is essentially the same as the HMM and v. 3: Viterbi algorithm.
- As with v. 3, learning is complicated and depends on the form of each “s.”
- This model strictly generalizes local classifiers (v. 0), the HMM (v. 2), and v. 3.

V. 4

Even better features for HMMs, with the promise of efficient decoding and learning.

Reflection

Claim: As we move from v. 1 (sequential classifiers) to v. 4 to v. 0 (local classifiers), the scoring functions available become strictly less expressive.



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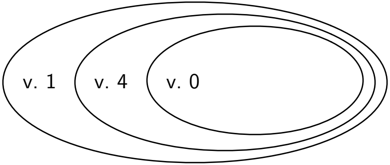

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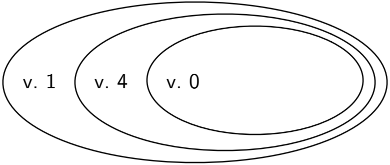

Compare v. 1 and v. 4. What kinds of features can you use in v. 1 that you can't use in v. 4?

Now consider v. 4 and v. 0. What kinds of features can you use in v. 4 that you can't use in v. 0?

Where We Are

	 0	 1	 2	 3	 4
Score decomp.	$s(\boldsymbol{x}, i, y_i)$	$s(\boldsymbol{x}, i, \boldsymbol{y}_{1:i})$	emission/ transition	$s(x_i, y_i)+$ $s(y_i, y_{i+1})$	$s(\boldsymbol{x}, i, y_i, y_{i+1})$
learn	SGD	?	count & normalize	?	?
decode	local	beam search	Viterbi	Viterbi	Viterbi

The Main Dish

Two Problems to Solve

1. Decoding: the Viterbi algorithm for choosing $\hat{\boldsymbol{y}}$.
   - Usually taught for classical HMMs (v. 2); I will teach it for v. 4, abstracting away "s."
2. Learning: estimating the parameters of each s function.
   - Depending on your choices here, you arrive at the structured perceptron, the classical conditional random field (CRF), neural CRFs, and more.

A Data Structure

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\dots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

The cell at row j, column i will hold information pertaining to choosing $\hat{y}_i = \ell_j$.

The End of the Sequence

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\ldots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

$$\begin{aligned} \hat{y}_n &= \underset{y_n \in \mathcal{L}}{\operatorname{argmax}} \sum_{i=0}^{n} s(\boldsymbol{x}, i, y_i, y_{i+1}) \\ &= \underset{y_n \in \mathcal{L}}{\operatorname{argmax}}\, s(\boldsymbol{x}, i, y_{n-1}, y_n) + s(\boldsymbol{x}, i, y_n, \bigcirc) \end{aligned}$$

The decision about $\hat{y}_n$ is a function of y_{n-1}, $\boldsymbol{x}$, and nothing else!

High-Level View of the Viterbi Algorithm

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- With a little bookkeeping, we can then trace backwards and recover the best label sequence.

Recurrence

First, think about the *score* of the best sequence.

Let $\heartsuit_i(y)$ be the score of the best label sequence for $\boldsymbol{x}_{1:i}$ that ends in y. It is defined recursively:

$$\heartsuit_{n+1}(\bigcirc) = \max_{y_n \in \mathcal{L}} s(\boldsymbol{x}, n, y_n, \bigcirc) + \boxed{\heartsuit_n(y_n)}$$

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\heartsuit_{n-1}(y) &= \max_{y_{n-2} \in \mathcal{L}} s(\boldsymbol{x}, n-2, y_{n-2}, y) + \boxed{\heartsuit_{n-2}(y_{n-2})}
\end{aligned}$$

Recurrence

First, think about the *score* of the best sequence.

Let $\heartsuit_i(y)$ be the score of the best label sequence for $\boldsymbol{x}_{1:i}$ that ends in y. It is defined recursively:

$$\begin{aligned}
\heartsuit_{n+1}(\text{⯃}) &= \max_{y_n \in \mathcal{L}} s(\boldsymbol{x}, n, y_n, \text{⯃}) + \boxed{\heartsuit_n(y_n)} \\
\heartsuit_n(y) &= \max_{y_{n-1} \in \mathcal{L}} s(\boldsymbol{x}, n-1, y_{n-1}, y) + \boxed{\heartsuit_{n-1}(y_{n-1})} \\
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&\vdots \\
\heartsuit_i(y) &= \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}
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&\vdots \\
\heartsuit_1(y) &= s(\boldsymbol{x}, 0, \bigcirc, y)
\end{aligned}
$$

Viterbi Procedure (Part I: Prefix Scores)

input sequence

		x_1	x_2	$\ldots$	x_n	
$\mathcal{L}$	ℓ_1					
	ℓ_2					
	$\vdots$					
	ℓ_L					
	🛑					

Viterbi Procedure (Part I: Prefix Scores)

input sequence

		x_1	x_2	$\ldots$	x_n	
$\mathcal{L}$	ℓ_1	$\heartsuit_1(\ell_1)$				
	ℓ_2	$\heartsuit_1(\ell_2)$				
	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$				
	⯃					

$$\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$$

Viterbi Procedure (Part I: Prefix Scores)

		input sequence				
		x_1	x_2	$\dots$	x_n	
$\mathcal{L}$	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$			
	ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$			
	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$	$\heartsuit_2(\ell_L)$			
	🛑					

$$\heartsuit_i(y) = \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}$$

Viterbi Procedure (Part I: Prefix Scores)

input sequence

		x_1	x_2	$\ldots$	x_n	
$\mathcal{L}$	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$		$\heartsuit_n(\ell_1)$	
	ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$		$\heartsuit_n(\ell_2)$	
	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$	$\heartsuit_2(\ell_L)$		$\heartsuit_n(\ell_L)$	
	⯃					

$$\heartsuit_n(y) = \max_{y_{n-1} \in \mathcal{L}} s(\boldsymbol{x}, n-1, y_{n-1}, y) + \boxed{\heartsuit_{n-1}(y_{n-1})}$$

Viterbi Procedure (Part I: Prefix Scores)

input sequence

		x_1	x_2	$\ldots$	x_n	
$\mathcal{L}$	ℓ_1	$\heartsuit_1(\ell_1)$	$\heartsuit_2(\ell_1)$		$\heartsuit_n(\ell_1)$	
	ℓ_2	$\heartsuit_1(\ell_2)$	$\heartsuit_2(\ell_2)$		$\heartsuit_n(\ell_2)$	
	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$	$\heartsuit_2(\ell_L)$		$\heartsuit_n(\ell_L)$	
	$\bigcirc$					$\heartsuit_{n+1}(\bigcirc)$

$$\heartsuit_{n+1}(\bigcirc) = \max_{y_n \in \mathcal{L}} s(\boldsymbol{x}, n, y_n, \bigcirc) + \boxed{\heartsuit_n(y_n)}$$

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Viterbi Procedure (Part I: Prefix Scores and Backpointers)

input sequence

		x_1	x_2	$\dots$	x_n	
	ℓ_1					
	ℓ_2					
$\mathcal{L}$	$\vdots$					
	ℓ_L					
	🛑					

Viterbi Procedure (Part I: Prefix Scores and Backpointers)

input sequence

		x_1	x_2	$\dots$	x_n	
$\mathcal{L}$	ℓ_1	$\heartsuit_1(\ell_1)$ $\mathrm{bp}_1(\ell_1)$				
	ℓ_2	$\heartsuit_1(\ell_2)$ $\mathrm{bp}_1(\ell_2)$				
	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$ $\mathrm{bp}_1(\ell_L)$				
	⯃					

$$\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$$

$$\mathrm{bp}_1(y) = \bigcirc$$

Viterbi Procedure (Part I: Prefix Scores and Backpointers)

input sequence

		x_1	x_2	$\ldots$	x_n	
	ℓ_1	$\heartsuit_1(\ell_1)$ $\mathrm{bp}_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $\mathrm{bp}_2(\ell_1)$			
	ℓ_2	$\heartsuit_1(\ell_2)$ $\mathrm{bp}_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $\mathrm{bp}_2(\ell_2)$			
$\mathcal{L}$	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$ $\mathrm{bp}_1(\ell_L)$	$\heartsuit_2(\ell_L)$ $\mathrm{bp}_2(\ell_L)$			
	⯃					

$$\heartsuit_i(y) = \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}$$

$$\mathrm{bp}_i(y) = \operatorname*{argmax}_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \boxed{\heartsuit_{i-1}(y_{i-1})}$$

Viterbi Procedure (Part I: Prefix Scores and Backpointers)

input sequence

		x_1	x_2	$\ldots$	x_n	
	ℓ_1	$\heartsuit_1(\ell_1)$ $\text{bp}_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $\text{bp}_2(\ell_1)$		$\heartsuit_n(\ell_1)$ $\text{bp}_n(\ell_1)$	
	ℓ_2	$\heartsuit_1(\ell_2)$ $\text{bp}_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $\text{bp}_2(\ell_2)$		$\heartsuit_n(\ell_2)$ $\text{bp}_n(\ell_2)$	
$\mathcal{L}$	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$ $\text{bp}_1(\ell_L)$	$\heartsuit_2(\ell_L)$ $\text{bp}_2(\ell_L)$		$\heartsuit_n(\ell_L)$ $\text{bp}_n(\ell_L)$	
	🛑					

$$\heartsuit_n(y) = \max_{y_{n-1}\in\mathcal{L}} s(\boldsymbol{x}, n-1, y_{n-1}, y) + \boxed{\heartsuit_{n-1}(y_{n-1})}$$

$$\text{bp}_n(y) = \operatorname*{argmax}_{y_{n-1}\in\mathcal{L}} s(\boldsymbol{x}, n-1, y_{n-1}, y) + \boxed{\heartsuit_{n-1}(y_{n-1})}$$

Viterbi Procedure (Part I: Prefix Scores and Backpointers)

input sequence

		x_1	x_2	$\dots$	x_n	
	ℓ_1	$\heartsuit_1(\ell_1)$ $\mathrm{bp}_1(\ell_1)$	$\heartsuit_2(\ell_1)$ $\mathrm{bp}_2(\ell_1)$		$\heartsuit_n(\ell_1)$ $\mathrm{bp}_n(\ell_1)$	
	ℓ_2	$\heartsuit_1(\ell_2)$ $\mathrm{bp}_1(\ell_2)$	$\heartsuit_2(\ell_2)$ $\mathrm{bp}_2(\ell_2)$		$\heartsuit_n(\ell_2)$ $\mathrm{bp}_n(\ell_2)$	
$\mathcal{L}$	$\vdots$					
	ℓ_L	$\heartsuit_1(\ell_L)$ $\mathrm{bp}_1(\ell_L)$	$\heartsuit_2(\ell_L)$ $\mathrm{bp}_2(\ell_L)$		$\heartsuit_n(\ell_L)$ $\mathrm{bp}_n(\ell_L)$	
	⯃					$\heartsuit_{n+1}(\text{⯃})$ $\mathrm{bp}_{n+1}(\text{⯃})$

$$\heartsuit_{n+1}(\text{⯃}) = \max_{y_n \in \mathcal{L}} s(\boldsymbol{x}, n, y_n, \text{⯃}) + \boxed{\heartsuit_n(y_n)}$$

$$\mathrm{bp}_{n+1}(\text{⯃}) = \operatorname*{argmax}_{y_n \in \mathcal{L}} s(\boldsymbol{x}, n, y_n, \text{⯃}) + \boxed{\heartsuit_n(y_n)}$$

Full Viterbi Procedure

Input: scores $s(\boldsymbol{x}, i, y, y')$, for all $i \in \{0, \ldots, n\}$, $y, y' \in \mathcal{L}$

Output: $\hat{\boldsymbol{y}}$

1. Base case: $\heartsuit_1(y) = s(\boldsymbol{x}, 0, \bigcirc, y)$
2. For $i \in \{2, \ldots, n+1\}$:
   - Solve for $\heartsuit_i(*)$ and $\mathrm{bp}_i(*)$.

$$\heartsuit_i(y) = \max_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \heartsuit_{i-1}(y_{i-1}),$$

$$\mathrm{bp}_i(y) = \operatorname*{argmax}_{y_{i-1} \in \mathcal{L}} s(\boldsymbol{x}, i-1, y_{i-1}, y) + \heartsuit_{i-1}(y_{i-1})$$

   (At $n+1$ we're only interested in $y =$ 🛑.)

3. $\hat{y}_{i+1} \leftarrow$ 🛑
4. For $i \in \{n, \ldots, 1\}$:
   - $\hat{y}_i \leftarrow \mathrm{bp}_{i+1}(\hat{y}_{i+1})$

Viterbi Asymptotics

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\dots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

Viterbi Asymptotics

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\ldots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

Space: need to store s, and fill in the cells above.

Viterbi Asymptotics

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\dots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

Space: need to store s, and fill in the cells above. $O(nL^2)$ for s (in the most general case, often less), $O(nL)$ for cells

Viterbi Asymptotics

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\ldots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

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Runtime: each cell requires an "argmax."

Viterbi Asymptotics

input sequence

labels in $\mathcal{L}$

	x_1	x_2	$\dots$	x_n
ℓ_1				
ℓ_2				
$\vdots$				
ℓ_L				

Space: need to store s, and fill in the cells above. $O(nL^2)$ for s (in the most general case, often less), $O(nL)$ for cells

Runtime: each cell requires an "argmax." $O(nL^2)$

Why it Works

Viterbi exploits the distributivity property:

$$\begin{aligned}
\max_{\boldsymbol{y}_{1:n}} \sum_{i=0}^{n} s(\boldsymbol{x}, i, y_i, y_{i+1}) &= \max_{y_n} s(\boldsymbol{x}, i, y_n, \bigcirc) + \max_{\boldsymbol{y}_{1:n-1}} \sum_{i=0}^{n-1} s(\boldsymbol{x}, i, y_i, y_{i+1}) \\
&= \max_{y_n} s(\boldsymbol{x}, i, y_n, \bigcirc) + \max_{y_{n-1}} s(\boldsymbol{x}, i, y_n - 1, y_n) \\
&\quad + \max_{\boldsymbol{y}_{1:n-2}} \sum_{i=0}^{n-2} s(\boldsymbol{x}, i, y_i, y_{i+1})
\end{aligned}$$

Max plus max plus max plus max plus . . .

Back to “s”

We haven’t said much about the function that scores candidate label pairs at different positions, $s(\boldsymbol{x}, i, y, y')$.

This function is very important; two common choices are:

- Expert-designed, task-specific features $\mathbf{f}(\boldsymbol{x}, i, y, y')$ and weights $\boldsymbol{\theta}$
- A neural network that encodes x_i in context, y_i, and y_{i+1} and gives back a goodness score

Either way, let $\boldsymbol{\theta}$ denote the parameters of s. From now on, we’ll use $s(\boldsymbol{x}, i, y, y'; \boldsymbol{\theta})$ and $\mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta})$ to emphasize that “s” is a function of parameters $\boldsymbol{\theta}$ we need to estimate.

Probabilistic View of Learning

As we've done before, we start with the principle of maximum likelihood to estimate $\boldsymbol{\theta}$:

$$\begin{aligned}
\boldsymbol{\theta}^* &= \arg\max_{\boldsymbol{\theta} \in \mathbb{R}^d} \prod_{i=1}^{T} p(\boldsymbol{Y} = \boldsymbol{y}_i \mid \boldsymbol{X} = \boldsymbol{x}_i; \boldsymbol{\theta}) \\
&= \arg\max_{\boldsymbol{\theta} \in \mathbb{R}^d} \sum_{i=1}^{T} \log p(\boldsymbol{Y} = \boldsymbol{y}_i \mid \boldsymbol{X} = \boldsymbol{x}_i; \boldsymbol{\theta}) \\
&= \arg\min_{\boldsymbol{\theta} \in \mathbb{R}^d} \sum_{i=1}^{T} \underbrace{-\log p(\boldsymbol{Y} = \boldsymbol{y}_i \mid \boldsymbol{X} = \boldsymbol{x}_i; \boldsymbol{\theta})}_{\text{sometimes called ``log loss'' or ``cross entropy''}}
\end{aligned}$$

Next, we'll drill down into "$p(\boldsymbol{Y} = \boldsymbol{y}_i \mid \boldsymbol{X} = \boldsymbol{x}_i; \boldsymbol{\theta})$."

Conditional Random Fields

Lafferty et al. (2001)

CRFs are a tremendously influential model that generalizes multinomial logistic regression to structured outputs like sequences.

$$
\begin{aligned}
p_{\mathrm{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta}) &= \frac{\exp \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta})}{Z(\boldsymbol{x}; \boldsymbol{\theta})} \\
Z(\boldsymbol{x}; \boldsymbol{\theta}) &= \sum_{\boldsymbol{y'} \in \mathcal{Y}(\boldsymbol{x})} \exp \mathrm{Score}(\boldsymbol{x}, \boldsymbol{y'}; \boldsymbol{\theta}) \\
-\log p_{\mathrm{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta}) &= -\underbrace{\mathrm{Score}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{\theta})}_{\text{“hope”}} + \underbrace{\log Z(\boldsymbol{x}; \boldsymbol{\theta})}_{\text{“fear”}}
\end{aligned}
$$

So, our “CRF”:

- Uses Viterbi for decoding (our v. 4 sequence labeler)
- Trains parameters to maximize likelihood (like MLR and NNs)

Conditional Random Field

Lafferty et al. (2001)

Sequence-Level Log Loss

Here's the maximum likelihood learning problem (equivalently, sequence-level log loss):

$$\boldsymbol{\theta}^* = \operatorname*{argmin}_{\boldsymbol{\theta} \in \mathbb{R}^d} \sum_{i=1}^{T} -\mathrm{Score}(\boldsymbol{x}_i, \boldsymbol{y}_i; \boldsymbol{\theta}) + \log Z(\boldsymbol{x}_i; \boldsymbol{\theta})$$

If we can calculate and differentiate (w.r.t. $\boldsymbol{\theta}$) the Score and Z functions, we can use SGD to learn.

Reflection

Given a training instance $\langle \boldsymbol{x}_i, \boldsymbol{y}_i \rangle$, what do you need to do to calculate $\mathrm{Score}(\boldsymbol{x}_i, \boldsymbol{y}_i; \boldsymbol{\theta})$?

Calculating $Z(\boldsymbol{x}; \boldsymbol{\theta})$

Good news! The algorithm that gives us Z is *almost exactly like* the Viterbi algorithm.

Forward algorithm: sums the $\exp \text{Score}$ values for all label sequences, given $\boldsymbol{x}$, in the same asymptotic time and space as Viterbi.

Let $\alpha_i(y)$ be the sum of all (exponentiated) scores of label prefixes of length i, ending in y.

Some Algebra

Given the decomposition

$$\mathrm{Score}(\boldsymbol{x},\boldsymbol{y};\boldsymbol{\theta}) = \sum_{i=0}^{n} s(\boldsymbol{x}, i, y_i, y_{i+1}; \boldsymbol{\theta}),$$

it holds that

$$\exp \mathrm{Score}(\boldsymbol{x},\boldsymbol{y};\boldsymbol{\theta}) = \prod_{i=0}^{n} e^{s(\boldsymbol{x}, i, y_i, y_{i+1}; \boldsymbol{\theta})},$$

and therefore

$$Z(\boldsymbol{x};\boldsymbol{\theta}) = \sum_{\boldsymbol{y'} \in \mathcal{Y}(\boldsymbol{x})} \prod_{i=0}^{n} e^{s(\boldsymbol{x}, i, y'_i, y'_{i+1}; \boldsymbol{\theta})}$$

Forward Algorithm

Input: scores $s(\boldsymbol{x}, i, y, y'; \boldsymbol{\theta})$, for all $i \in \{0, \dots, n\}$, $y, y' \in \mathcal{L}$

Output: $Z(\boldsymbol{x}; \boldsymbol{\theta})$

1. Base case: $\alpha_1(y) = e^{s(\boldsymbol{x}, 0, \bigcirc, y; \boldsymbol{\theta})}$
2. For $i \in \{2, \dots, n+1\}$:
   - Solve for $\alpha_i(*)$.

     $$\alpha_i(y) = \sum_{y_{i-1} \in \mathcal{L}} e^{s(\boldsymbol{x}, i-1, y_{i-1}, y; \boldsymbol{\theta})} \times \alpha_{i-1}(y_{i-1})$$

     (At $n+1$ we're only interested in $y = \text{⯃}$.)
3. Return $\alpha_{n+1}(\text{⯃})$, which is equal to $Z(\boldsymbol{x}; \boldsymbol{\theta})$.

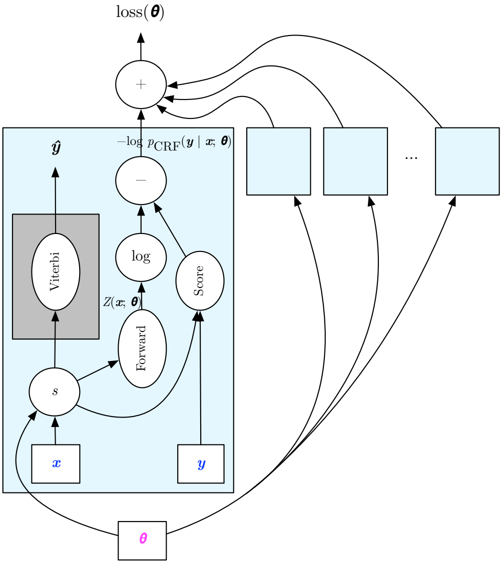
Intuitions about the Forward Algorithm

Just as Viterbi changes “scary max over big sum” to “max plus max plus max plus . . . ,”
the Forward algorithm changes “scary sum over big product” to “plus times plus times plus times . . . .”

If you organize the operations in the other direction, you get the Backward algorithm.

You can differentiate Z with respect to s, because it’s all just $\exp$, addition, and multiplication. If you mechanically derive the partial derivatives, you will rediscover the Backward algorithm.

Computation Graph View of CRF



$\text{loss}(\boldsymbol{\theta})$
$+$
$-\log p_{\text{CRF}}(\boldsymbol{y} \mid \boldsymbol{x}; \boldsymbol{\theta})$
$\hat{\boldsymbol{y}}$
$-$
log
Viterbi
Score
$Z(\boldsymbol{x}; \boldsymbol{\theta})$
Forward
s
$\boldsymbol{x}$
$\boldsymbol{y}$
...
$\boldsymbol{\theta}$

Reflection

Earlier in the lecture, I promised that learning would have some guarantees. Consider:

- The runtime and space requirements for calculating the loss and gradient, as a function of the data.
- The conditions under which we can confidently expect convergence to a global optimum of the likelihood if we use SGD.

An Alternative: Structured Perceptron

Recall that CRF = v. 4 + sequence-level log loss.

Perceptron loss (Collins, 2002):

$$\boldsymbol{\theta}^* = \operatorname*{argmin}_{\boldsymbol{\theta} \in \mathbb{R}^d} \sum_{i=1}^{T} -\mathrm{Score}(\boldsymbol{x}_i, \boldsymbol{y}_i; \boldsymbol{\theta}) + \max_{\boldsymbol{y}} \mathrm{Score}(\boldsymbol{x}_i, \boldsymbol{y}; \boldsymbol{\theta})$$

The structured perceptron = v. 4 + perceptron loss.

Regularization

Just as in classification with linear and non-linear models, you'll want to take steps to avoid overfitting.

The same tools (e.g., ℓ_2 and ℓ_1 penalties for linear model weights, and dropout for neural networks) can be used here.

Digestif: Connections and Generalizations

V. 2–4 are weighted finite-state machines (think of labels as states).

The models we saw today are all "first order" sequence models in the sense that each y_i only interacts with one immediate neighbor through s.

- Second-order: $\text{Score}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=0}^{n} s(\boldsymbol{x}, i, y_i, y_{i+1}, y_{i+2})$
- mth-order: $\text{Score}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=0}^{n} s(\boldsymbol{x}, i, \boldsymbol{y}_{i:i+m})$

Viterbi for mth order has $O(nL^{m+1})$ runtime.

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