Natural Language Processing (CSE 517): Weighted Finite-State Transducers

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Winter 2023

Readings: Eisenstein (2019) 9.0–9.1

Motivation

- Dominant perspective in NLP in the 1970s–80s: formal language theory
- Engineering approach: expert-crafted, formally constrained, purely symbolic systems
- Relevance today: computational models of morphology

Morphology

Extensive overview: Bender (2013)

race $\rightarrow$ races
race $\rightarrow$ racing
race $\rightarrow$ raced

Morphology

Extensive overview: Bender (2013)

grace $\rightarrow$ graceful
graceful $\rightarrow$ gracefully
grace $\rightarrow$ disgrace
disgrace $\rightarrow$ disgraceful
disgraceful $\rightarrow$ disgracefully
friend $\rightarrow$ unfriend
Obama $\rightarrow$ Obamacare

Morphology

Extensive overview: Bender (2013)

uygarlaştıramadıklarımızdanmışsınızcasına

"(behaving) as if you are among those whom we could not civilize"

Reflection

What (natural) languages do you know? What are some examples of the morphology in those languages?

Aperitif: Finite-State Automata

A finite-state automaton (plural “automata”) consists of :

- a finite alphabet of input symbols, Σ
- a finite set of states, Q
- a start state, $q_0 \in Q$
- a set of final states, $F \subseteq Q$
- a transition function that maps a state and a symbol (or an empty string, denoted ε) to a set of states, $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow 2^Q$

We visualize an FSA with a state diagram.

Aperitif: Finite-State Automata

A finite-state automaton (plural "automata") consists of (toy example in blue):

- a finite alphabet of input symbols, Σ $\qquad \Sigma = \{a, b\}$
- a finite set of states, Q $\qquad Q = \{q_0, q_1\}$
- a start state, $q_0 \in Q$ $\qquad q_0$
- a set of final states, $F \subseteq Q$ $\qquad F = \{q_1\}$
- a transition function that maps a state and a symbol (or an empty string, denoted ε) to a set of states, $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to 2^Q$

$$\delta = \left\{ \begin{array}{ccc} (q_0, a) & \to & \{q_0\}, \\ (q_0, b) & \to & \{q_1\}, \\ (q_1, a) & \to & \emptyset, \\ (q_1, b) & \to & \{q_1\} \end{array} \right\}$$

We visualize an FSA with a state diagram.

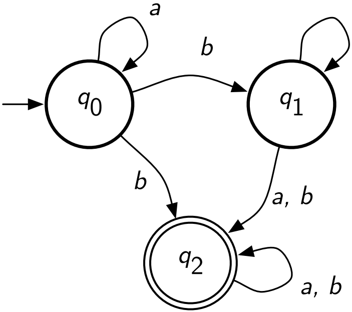
State Diagram for our Toy Example FSA

- $\Sigma = \{a, b\}$
- $Q = \{q_0, q_1\}$
- $F = \{q_1\}$
- $\delta = \left\{ \begin{array}{ccc} (q_0, a) & \rightarrow & \{q_0\}, \\ (q_0, b) & \rightarrow & \{q_1\}, \\ (q_1, a) & \rightarrow & \emptyset, \\ (q_1, b) & \rightarrow & \{q_1\} \end{array} \right\}$

FSAs and their Languages

- A language is a set of strings; for FSA $\mathcal{F}$ we denote by $L(\mathcal{F})$ the set of strings it accepts.
- Regular languages: the set of languages recognizeable by FSAs.
- A *path* through the FSA $\mathcal{F}$ serves as a proof that the path's string is in $L(\mathcal{F})$.
- An FSA is *deterministic* (a "DFA") if there is exactly one path per string in $L(\mathcal{F})$.
- Given DFA $\mathcal{F}$ and a string of length n, we can check membership in $L(\mathcal{F})$ in $O(n)$ time and $O(1)$ space.

Nondeterministic FSA

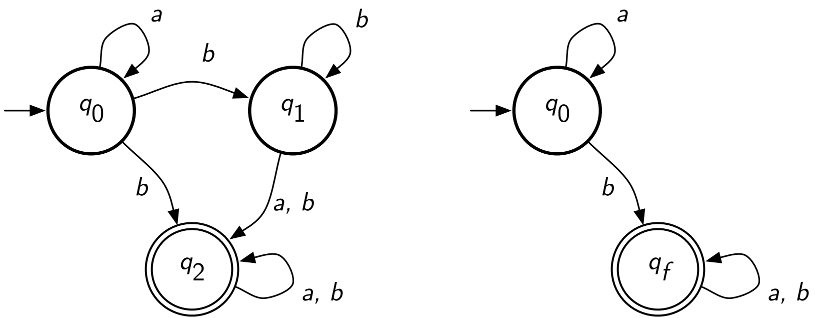


Accepts any string of as and bs that includes at least one b.

Some Theoretical Properties of Regular Languages

- Closed under intersection, union, subtraction, concatenation, negation, Kleene closure, reversal, and more operations.
- There things they cannot do! E.g., counting. $a^n b^n$ is not a regular language. The pumping lemma is a formal tool used to prove that a language is not regular.
- Any nondeterministic FSA can be mechanically transformed into a deterministic one with the same language, but the number of states may explode.

NDFA and DFA with the same language.



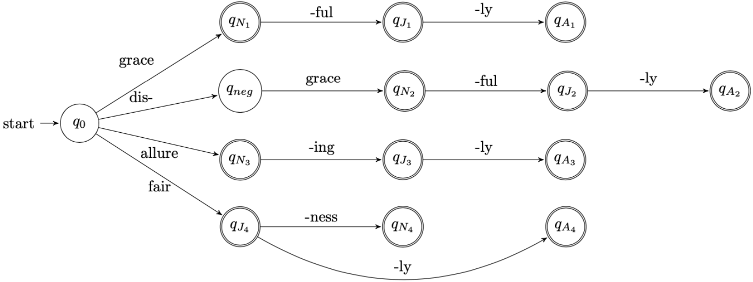
How can we use it?

"Vocabulary machine": an FSA whose language includes all (and only) the words in a language (e.g., English). (Σ is the set of characters used in the language.)

Advantage over a simple brute-force list: encode rules that let us generate new words (e.g., *Clintonian*, *Trumpism*, *coronafuckingvirus*).

Example

Eisenstein (2019) figure 9.2 (p. 187)



Adding Weights

A powerful generalization is the **weighted** FSA (WFSA), which augments every path with a score. A WFSA consists of:

- a finite alphabet of input symbols, Σ
- a finite set of states, Q
- an initial weight function, $\lambda : Q \rightarrow \mathbb{R}$
- a final weight function, $\rho : Q \rightarrow \mathbb{R}$
- a transition function that weights maps a state pair and a symbol (or ε), $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times Q \rightarrow \mathbb{R}$

Reflection

Can you show how an unweighted FSA is a special case of a WFSA? Hint: imagine that there are only two values that λ, ρ, and δ can map to, 0 and $-\infty$.

Scoring a Path

Consider a path of n transitions, $q_0 \stackrel{x_1}{\rightarrow} q_1 \stackrel{x_2}{\rightarrow} q_2 \cdots q_{n-1} \stackrel{x_n}{\rightarrow} q_n$.

The score of the path is given by

$$\lambda(q_0) + \left(\sum_{i=1}^{n} \delta(q_{i-1}, x_i, q_i) \right) + \rho(q_n)$$

You can think of weights as "costs" and imagine trying to find the minimum-cost path through a WFSA for a given string $\boldsymbol{x}$.

Reflection

Can you think of a good use for "costs" or scores associated with the words in our vocabulary machine's language?

The Main Dish

Weighted Finite-State Transducers

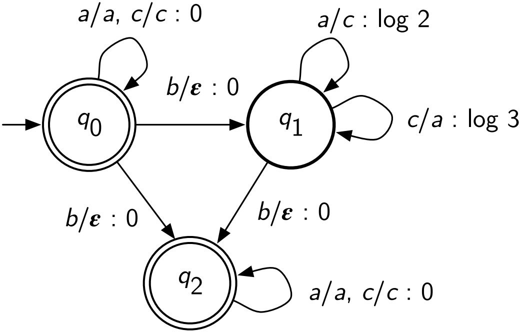
WFSTs encode weighted *relations* between strings. They consist of:

- a finite alphabet of input symbols, Σ
- **a finite alphabet of output symbols,** Ω
- a finite set of states, Q
- an initial weight function, $\lambda : Q \to \mathbb{R}$
- a final weight function, $\rho : Q \to \mathbb{R}$
- a transition function that weights maps a state pair and a **pair of symbols** (or ε), $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Omega \cup \{\varepsilon\}) \times Q \to \mathbb{R}$

Reflection

WFSTs generalize unweighted FSTs, WFSAs, and unweighted FSAs!
Can you sketch out a way to convert any of those into a WFST?

Example of a WFST

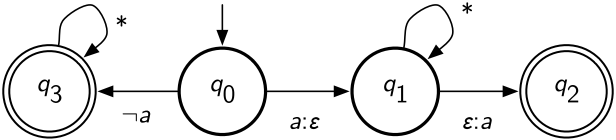


Properties of WFSTs

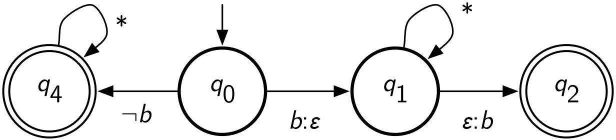
- If you strip away either the inputs or the outputs, you get a WFSA and the language is regular.
- Most important property: WFSTs are closed under composition. Consider the unweighted case.
  - Let $\mathcal{F}$ be an FST encoding pairs $F \subseteq \Sigma^* \times \Gamma^*$.
  - Let $\mathcal{G}$ be an FST encoding pairs $G \subseteq \Gamma^* \times \Omega^*$.
  - Then $\mathcal{G} \circ \mathcal{F}$ denotes $\{(\boldsymbol{x}, \boldsymbol{z}) \mid \exists \boldsymbol{y} \in \Gamma^*, (\boldsymbol{x}, \boldsymbol{y}) \in F \wedge (\boldsymbol{y}, \boldsymbol{z}) \in G\}$.
  - There is an FST that encodes $\mathcal{G} \circ \mathcal{F}$.

Illustrating Composition

$\mathtt{F}$ maps $a\alpha$ to αa and $(\neg a)\alpha$ to itself:

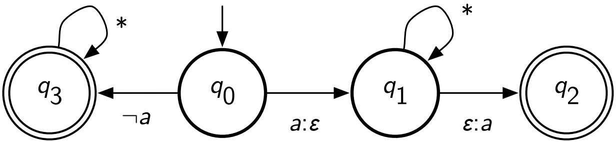


$\mathtt{G}$ maps $b\alpha$ to αb and $(\neg b)\alpha$ to itself:

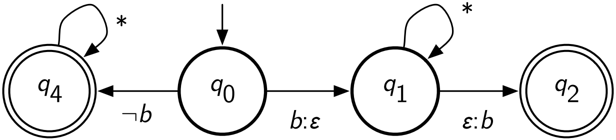


Illustrating Composition

F maps $a\alpha$ to αa and $(\neg a)\alpha$ to itself:



G maps $b\alpha$ to αb and $(\neg b)\alpha$ to itself:



We can implement both $G \circ F$ and $F \circ G$ by applying FST composition, and both will be FSTs.

Illustrating Composition

input	output of . . . F	G ∘ F	G	F ∘ G
abc				
bad				
def				

Illustrating Composition

input	output of . . .			
	F	G ∘ F	G	F ∘ G
abc	*bca*			
bad				
def				

Illustrating Composition

input	output of . . . F	$G \circ F$	G	$F \circ G$
abc	*bca*	*cab*		
bad				
def				

Illustrating Composition

input	output of … F	G ∘ F	G	F ∘ G
abc	*bca*	*cab*	*abc*	
bad				
def				

Illustrating Composition

input	output of ...			
	F	G ∘ F	G	F ∘ G
abc	*bca*	*cab*	*abc*	*bca*
bad				
def				

Illustrating Composition

input	output of . . .			
	$\mathtt{F}$	$\mathtt{G} \circ \mathtt{F}$	$\mathtt{G}$	$\mathtt{F} \circ \mathtt{G}$
abc	*bca*	*cab*	*abc*	*bca*
bad	*bad*	*adb*	*adb*	*dba*
def				

Illustrating Composition

input	output of . . .			
	F	G ∘ F	G	F ∘ G
abc	*bca*	*cab*	*abc*	*bca*
bad	*bad*	*adb*	*adb*	*dba*
def	*def*	*def*	*def*	*def*

(W)FST as a Declarative System

For convenience, we talk about an "input" and an "output" string, but the same model can also be thought of as:

- Mapping output strings to input strings
- Recognizing pairs of strings
- Generating pairs of strings

You should think of FSTs as primarily a *declarative* framework (not a procedure).

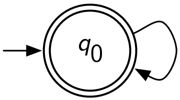
Avoid this confusion: FSTs are *not* functions from inputs to outputs; an input string can pair with more than one output string (and vice versa).

Putting WFSTs to Work

Levenshtein edit distance: what's the minimum number of single-character deletions, insertions, or substitutions to change x into x'?

You only need one state! The classic dynamic programming algorithm emerges when you apply conventional shortest-path algorithms.

Levenshtein Distance WFST, $\Sigma = \{a, b\}$



$\lambda(q_0) = \rho(q_0) = 0$

$a/a : 0$
$b/b : 0$ — no change, no cost

$a/\varepsilon : 1$
$b/\varepsilon : 1$ — deletions cost 1

$\varepsilon/a : 1$
$\varepsilon/b : 1$ — insertions cost 1

$a/b : 1$
$b/a : 1$ — substitutions cost 1

Putting WFSTs to Work

Problem: surface variation in words hides semantic (near) equivalence. E.g., the subtle differences among {*invite*, *invited*, *inviting*, *invites*} do not matter for many applications.

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Porter (1980) stemmer: an algorithm that strips suffixes from English words (without "knowing" any words) according to a set of rules, such as:

$$\textit{-sses} \rightarrow \textit{-ss}$$
$$\textit{-ies} \rightarrow \textit{-i}$$
$$\textit{-ss} \rightarrow \textit{-ss} \text{ OR } \textit{-s} \rightarrow \varepsilon$$

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Stemming lets a system abstract away from words a bit, so that (e.g.) a search engine query for *parties where cats are invited* will match documents with *invite a cat to a party* as well.

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(Today, people use data-driven methods like byte-pair encoding (Sennrich et al., 2016) to segment words into pieces, sometimes called "wordpieces.")

What about today?

Finite-state transducers (sometimes weighted, sometimes not) are arguably the best way to encode the morphological systems of many languages.

Goal: map between words we see in text ("surface" forms) and morphological analyses into a lemma or base/root form of the word plus "features."

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Example from Spanish (surface ↔ analysis):

canto	↔	*cantar*+Verb+PresentIndicative+1stPerson+Singular
como	↔	*comer*+Verb+PresentIndicative+1stPerson+Singular
comes	↔	*comer*+Verb+PresentIndicative+2ndPerson+Singular

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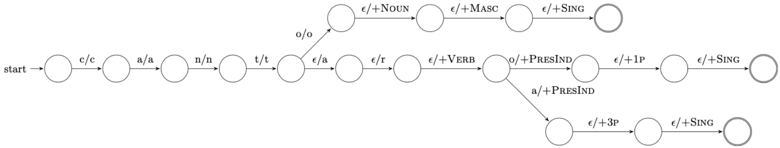
como	$\leftrightarrow$	*comer*+Verb+PresentIndicative+1stPerson+Singular
comes	$\leftrightarrow$	*comer*+Verb+PresentIndicative+2ndPerson+Singular

If you use a (W)FST, you can invert input and output and use the same model for analysis and generation!

Example

Eisenstein (2019) figure 9.7 (p. 195)



Designing an FST for Morphology: Notes

The challenge is to avoid both under- and over-generation. E.g., we want *feet*/*foot*+Plural and *beets*/*beet*+Plural, but not *foots*/*foot*+Plural or *beet*/*boot*+Plural!

Designing an FST for Morphology: Notes

Because FSTs encode relations, we can elegantly handle optionality (e.g., *colours*/*color*+Noun+Plural and *colors*/*color*+Noun+Plural) and ambiguity (e.g., *bears*/*bear*+Noun+Plural and *bears*/*bear*+Verb+Present+3rdPerson+Singular).

Designing an FST for Morphology: Notes

Because of closure under composition, union, concatenation, etc., you can buid separate modules for different morphology rules, or parts of the vocabulary.

Designing an FST for Morphology: Notes

Usually some parts are "lexicons" or FSTs that encode sets of words to which the same rules are applied (e.g., "*-er* verbs" in French).

Designing an FST for Morphology: Notes

It's hard to avoid the writing system (orthography) of a language; some of your rules will probably be more about writing conventions than the language as it is spoken. E.g., English past tense adds *-ed* to a verb's base form, but in the writing system we don't do this if the word ends in silent *e*: *bake* becomes *baked*, not *bakeed*.

Linguistic Note

Examples curated by Fokkens (2009)

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- Transfixation, e.g., Maltese has the root *ktb*, from which are formed words like *kiteb* ("he wrote"), *kitbu* ("they wrote"), *miktub* ("written"), *ktieb* ("book"), *kotba* ("books"), and more (Crysmann, 2006)

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- Subtraction, e.g., Koasati has singular *pitaf-fi-n* and plural *pit-li-n* ("to slice up in the middle") and *acokcana:-kaln* singular and *acokcan-ka-n* plural ("to quarrel with someone") (Sproat, 1992)

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- Reduplication, e.g., Indonesian has *orang* ("man") and *orang orang* ("men") (Crysmann, 2006)

Notable NLP Tools

- Foma: https://fomafst.github.io (designed for manual programing of FSTs; Hulden, 2009; see also Beesley and Karttunen, 2003)
- OpenFST: http://www.openfst.org/ (designed for WFST operations)
- EpiTran: grapheme-to-phoneme conversion for lots of languages (Mortensen et al., 2018)

Reflection

The Yiddish language is conventionally written in a variant of the Hebrew alphabet, but it can also be transliterated into the Latin alphabet we use for English. The former is written right-to-left, the latter left-to-right.

Assuming we keep characters in the order they appear on a printed page, could we use a (W)FST to map Yiddish words in either alphabet into the other?

Cautionary Note

A computational model of a natural language's morphology probably encodes:

- The rules as known to one particular community of speakers (often a privileged one)
- Orthographic conventions of one such community

But a language (and writing) vary a lot across communities of its users.

Ask: who was/is this system built for?

Digestif: Remarks

Current NLP research is not very focused on finite-state methods, but they are worth knowing about because:

- For some language problems, you can manually program a nearly perfect solution if you choose the right formalism and work hard for good coverage.
- Morphology is a huge challenge in some languages; the number of possible words can be large, and many won't appear in text collections.
- Later, you'll hear me say that some methods are "uninterpretable" and "not formally understood." WFSTs are the opposite of that!

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